

AN ANALYSIS OF THE ADAPTIVE CLUSTER SAMPLING DESIGN WITH RARE
PLANT POINT DISTRIBUTIONS

By

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We certify that we have read this study and that it conforms to acceptable standards of scholarly presentation and is fully acceptable, in scope and quality, as a thesis for the degree of Master of Arts.

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ABSTRACT

A sampling design that can provide estimates of abundance with low variance is very valuable to biologists working with limited budgets and time. Estimates that are precise even with low sampling efforts allow researchers to cheaply and confidently monitor rare populations. Adaptive cluster sampling has the potential to be much more efficient at sampling rare populations than conventional sampling designs, but it has also been shown to be highly inappropriate for some populations. Applications of adaptive cluster sampling (ACS) have had inconsistent results in real-world settings, leading to increasing scrutiny of the factors that influence the efficiency of this design. Much more work still needs to be done in order to provide samplers with the knowledge of when ACS is appropriate and how to maximize its effectiveness through constructing an optimal design. This study develops a procedure in a GIS environment for rigorously examining the effects of design parameters on the variance of ACS estimates, and applies this procedure to some real-world point populations. The relative efficiency of adaptive cluster sampling to simple random sampling is shown to be dramatically influenced by design parameters. This highlights the need for further investigation and a better understanding of how these parameters interact with point distributions through the use of procedures and tools such as those introduced here.

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INTRODUCTION

In ecological studies, efficiently sampling rare species is a difficult challenge. Conventional sampling designs may require substantial effort in order to achieve even moderate precision (e.g. Evans and Viengkham 2001). Unfortunately, accurate estimates of abundance are often most needed for rare species. Adaptive cluster sampling is attractive because it can perform more efficiently than conventional designs for geographically rare, clustered populations. It allows biologists to survey larger fractions of the target population while providing design-unbiased estimates. Unfortunately, the adaptive cluster sampling design is not appropriate for all populations, and has sometimes resulted in excessive survey costs for very little benefit (Smith et al. 2003). While many articles on adaptive cluster sampling have been published since it was first described by Thompson (1990), accounts describing direct applications of the design have been limited. What's more, these applied studies show a remarkable variability in the performance of adaptive cluster sampling in field settings (Smith et al. 2004). It is apparent that much more work is needed to provide guidelines that will allow biologists to realize the potential of this design.

Adaptive cluster sampling (ACS) was proposed by Thompson (1990) specifically with rare, clustered populations in mind. The ACS design calls for an initial sample of n_1 units to be drawn from a defined universe (a finite set of basic sampling units) according to a conventional sampling design, such as simple random sampling (SRS). It then operates under the rule that if any of these initially selected units satisfies a certain

condition of interest, C , additional units in the neighborhood of that unit will be added to the sample. In area-based sampling, neighborhoods are usually defined based on spatial proximity, such as all units sharing an edge with the initially selected unit. In sampling biological populations, the condition to include additional units in a sample (C) is usually based on the count of individuals of the target population within the initial sample of units. Thus, any initially sampled units containing enough individuals to satisfy the condition C will cause additional units in their neighborhood to be added to the sample. If any of these additional units also satisfy C , further sampling of their neighborhoods occurs as well. This process continues until no further neighborhood units satisfy C . Through this process, ACS takes advantage of clustering within a population to make units containing interesting information more likely to be included in the sample. For example, in a sample of a spatially clustered population (Fig. 1) the condition C could be simply the presence of an individual ($y_i > 0$), and any occupied quadrats in the initial sample would add neighboring quadrats until only empty units were encountered. The result is a set of *clusters*, each comprised of a core of units that satisfy C and an outer layer of edge units that do not. If all units in the realized adaptive sample received equal weight, conventional equal probability abundance estimators would be positively biased. However, Thompson (1990) showed that modified versions of the Horvitz-Thompson and Hansen-Hurwitz estimators could account for the unequal probabilities of selection imposed by the adaptive design. These estimators are design-unbiased (they are unbiased without relying on any assumptions about the population).

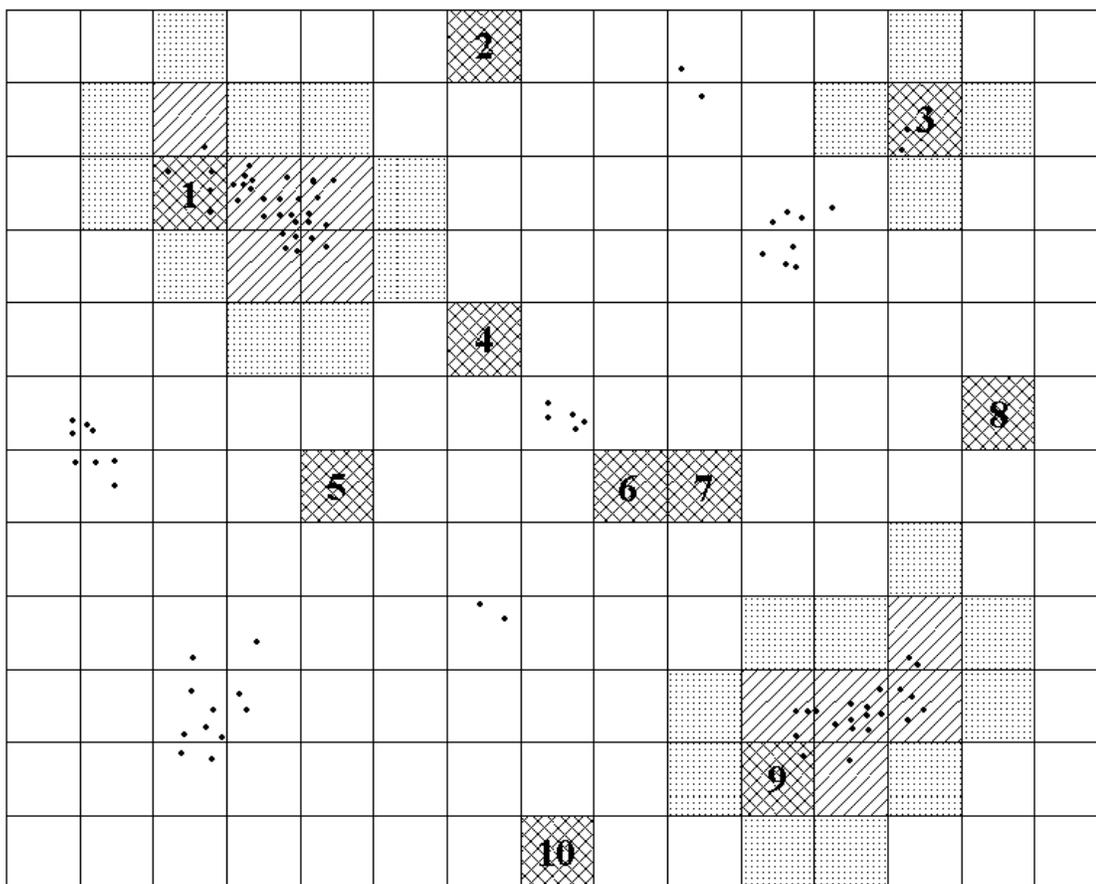


Figure 1. An example of an adaptive cluster sample. Numbered units are part of the initial sample, additional units that satisfy C are indicated with diagonal lines, and empty edge units are stippled.

Much theoretical work was done with the ACS design subsequent to its introduction in 1990, but very few direct applications were performed in biological settings until the last decade. While some recent studies have found ACS to live up well to its promising potential, the majority of studies have had either mixed or fully negative results regarding the applicability of ACS to real biological systems. Acharya et al.

(2000) sampled rare tree species in Nepal and found ACS to work well for some but not for others, concluding that the applicability had to do with distribution characteristics of the different species (some were more clustered than others). Magnussen et al. (2005), who used ACS to estimate deforestation rates, and Hanselman et al. (2003) who used it to sample rockfish off Alaska found that it provided more precise estimates, but these gains in precision were offset by increasing costs due to adaptively adding sampling units, especially edge units. ACS has performed poorly for other researchers. A study by Morrison et al. (2008), comparing sampling designs with an aggregated winter annual plant, led them to conclude that the population's distribution was simply inappropriate for ACS. Smith et al. (2003), Noon et al. (2006), and Goldberg et al. (2007) all acknowledged the fact that ACS was better at detecting a higher fraction of individuals within the population, but found that it uniformly failed to provide more precise estimates of population size.

The specific problems that samplers have had with ACS are in general due to either excessive realized sample sizes, a disproportionately high fraction of the sample being edge units, or bias of estimates resulting from the use of devices to curb excessive sample sizes. These general problems have been for the most part anticipated or duly noted by theoreticians over the years, and a number of articles have been published offering modifications of either the original basic design or estimators. Because the final size of the realized adaptive sample is unknown, it can be difficult to control the total sampling effort and accurately plan the cost of a survey in advance. Christman and Lan (2001) proposed an inverse ACS design meant to reduce final sample size. Brown and

Manly (1998) described a restricted ACS design that allows a preset limit on the final sample size to be imposed on ACS, and Salehi and Seber (2002) put together an unbiased estimator for this design. Thompson (1996) discussed a design that allows researchers to control the number of units added to a sample by ordering the values of the completed initial sample and using a certain percentile to choose the condition to further add units to the sample. Unfortunately, it is not always possible in real-world situations to completely finish the initial sample before adding units adaptively. Another downside of the ACS design is that the information from edge units is not incorporated into estimators unless they are encountered in the initial sample. Thus the ratio of edge units to network units can have a strong effect on the efficiency of ACS. Thompson and Seber (1996), Salehi (1999), and Dryver & Thompson (2005) have used what is known as the Rao-Blackwell method to develop estimators that utilize edge units, but are more complicated to calculate.

While most applications of ACS have been ineffective in some way, there have also been multiple examples of success. In an inventory of sparse forest populations, Talvitie et al. (2006) found ACS to be “considerably more effective” than SRS. Philippi (2005) successfully used ACS to estimate the abundance of local populations of low-abundance plants, Skibo et al. (2008) employed a modified ACS design to efficiently sample red sea urchin populations and Sullivan et al. (2008) found ACS to work well for a large proportion of the sea lamprey populations they examined. These successes have been predicted in multiple simulation studies as well (Christman 1997, Christman & Pontius 2000). In these examples, performance has typically been based on the precision

of estimates provided by ACS relative to those from other, more conventional designs given an equal effort. Regardless of the precision of estimates, a frequently acknowledged benefit provided by ACS is its tendency to sample a higher fraction of occupied sample units, allowing additional information to be collected concurrent to the sample (Lo et al. 1997, Noon et al. 2006, Smith et al. 2003). This can be especially useful when studying rare or endangered species for which such additional information is frequently lacking.

Overall there seems to be a gap between the theoretical potential of ACS and its realized performance in the field. In part, this is due to a characteristic of the ACS design that can be considered both an advantage and disadvantage. While ACS is in general recommended for rare, clustered populations, samplers have a considerable degree of flexibility in constructing a specific sample, potentially allowing it to be tailored to a wide variety of different situations and distributions. At the same time, it is the existence of so many different options that makes determining an optimal strategy so difficult. There are multiple alternatives for neighborhood type, the condition C , the size of the initial sample, the initial method of selecting the sample, the size of the basic sampling unit, and the estimator that utilizes the collected information. There have been several neighborhood definitions described for ACS (see Christman 2000) but the most common definition used in grid-based sampling is the first-order neighborhood, which includes the four immediately adjacent quadrats. Setting an appropriate C can be very important in designing an efficient adaptive cluster sample. Hanselman et al. (2003) determined retrospectively that had they used a more restrictive C they would have done substantially

less sampling with very little loss of estimation precision. The initial sample size (n_i) must be set high enough to ensure the inclusion of some networks, but if set too high it may lead to excessive sizes of the realized adaptive sample. There are also a number of different options for selection of the initial sample, such as simple random sampling with or without replacement (Thompson 1990), strip sampling, systematic sampling (Thompson 1991a), stratified sampling (Thompson 1991b) and simple Latin square sampling (Borkowski 1999). Both the modified Hansen-Hurwitz and Horvitz-Thompson estimators suggested by Thompson (1990) have seen use in applied situations, and both have had advantages and disadvantages identified with them (Phillipi 2005, Salehi 2003).

Researchers have also consistently noted that the efficiency of ACS is dependant upon the spatial distribution patterns of the particular population being studied, but surprisingly little attention has been paid to sample design parameters that are fully under the control of samplers and significantly effect the distribution of point populations across grid-based sampling universes. In particular, White (2004) showed that for members of the *Astragalus* genus, an herbaceous member of the plant family Fabaceae, distributions in a grid-based setting can be highly influenced by both the placement of the grid and the size of the grid cells. Weigand (2007) found species to exhibit clustering at multiple spatial scales, and random distributions at other scales. Thus by coordinating the size of the basic sampling unit with the scale of clustering within the target population researchers can have a great deal of control over the effectiveness of ACS.

The first aim of this study is to develop and introduce a procedure that allows for the rigorous examination of the effects of a) size of basic sample units, b) size of the initial sample (sampling fraction) and c) the condition C on the performance of ACS with both the Hansen-Hurwitz and Horvitz-Thompson estimators. The second aim is to apply this procedure to real-world plant populations in order to explore the influence that design parameters can have on the efficiency of ACS relative to Simple Random Sampling.

METHODS

Notation and Estimators

This study makes use of the modified versions of the Hansen-Hurwitz (HH) and Horvitz-Thompson (HT) estimators originally proposed by Thompson (1990) and compares them to the Simple Random Sample (SRS) estimator by calculating their relative efficiencies (Thompson & Seber 1996) at equal sample sizes.

In simple random sampling, the estimator of the population total (\hat{T}) given a random sample of n primary units from N total units is the sum of the y -values associated with each unit i included in the sample, divided by the probability that any unit i is included in the sample (π_i). In this case the y -values will represent the number of individuals within each quadrat. Because each unit is equally likely to be included, π_i is the same for all units and is equivalent to the fraction of the total sample space being sampled (n/N). The SRS estimator of the population total can be written as

$$\hat{T}_{SRS} = \frac{N}{n} \sum_{i=1}^n y_i \quad (1)$$

The sampling variance of this estimator can be calculated as

$$V(\hat{T}_{SRS}) = N(N-n) \frac{\sigma^2}{n} \quad (2)$$

where σ^2 is the finite population variance,

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \mu)^2 \quad (3)$$

and μ is the population mean, or the average number of individuals per unit,

$$\mu = \frac{1}{N} \sum_{i=1}^N y_i \quad (4)$$

Because inclusion probabilities for the units selected in an adaptive cluster sample are not all equal, conventional estimators such as \hat{T}_{SRS} are biased. Thompson (1990) suggested two design-unbiased estimators based on the Horvitz-Thompson (HT) and Hansen-Hurwitz (HH) estimators. Though past studies indicate that the HH estimator usually has a higher variance than the HT estimator, the HH estimator has seen as much if not more use in the field due to the complicated and laborious calculations associated with the HT estimator.

The true Horvitz-Thompson estimator is based on unit inclusion probabilities (π_i), but with an adaptive cluster sample it is impossible to know the inclusion probabilities for every basic sampling unit that is selected. However, it is possible to know the probability of including a *network* in the sample. A network is a subset of the units found in a cluster, such that selection of any unit within the network would lead to the inclusion of all other units in the network. Units that don't satisfy C but are in the neighborhood of one that does are known as *edge units*. Thus, a cluster with its edge units removed is a network. In addition, all units that fail to satisfy the condition C (including both edge and non-edge units) are considered networks of size one.

When an adaptive cluster sample is partitioned into distinct networks instead of basic sampling units, the Horvitz-Thompson estimator for the population total can be modified and expressed as

$$\hat{T}_{HT} = \sum_{k=1}^K \frac{y_k^* z_k}{\alpha_k} \quad (5)$$

Where y_k^* is the total number of individuals in the k^{th} network, K is the number of distinct networks in the sample, and α_k is the probability of including any unit in network k . If there are x_k units in the k^{th} network, then

$$\alpha_k = 1 - \frac{\binom{N - x_k}{n_1}}{\binom{N}{n_1}} \quad (6)$$

The sampling variance of this estimator is

$$V(\hat{T}_{HT}) = \sum_{k=1}^K \sum_{h=1}^K \frac{y_k^* y_h^* (\alpha_{kh} - \alpha_k \alpha_h)}{\alpha_k \alpha_h} \quad (7)$$

where α_{kh} is the probability of including both network k and network h in the adaptive sample and is defined as

$$\alpha_{kh} = 1 - \left[\frac{\binom{N - x_k}{n_1} + \binom{N - x_h}{n_1} - \binom{N - x_k - x_h}{n_1}}{\binom{N}{n_1}} \right] \quad (8)$$

The true Hansen-Hurwitz estimator is based on the probability of selecting a primary sampling unit on any given draw. Just as with inclusion probabilities though, draw-by-draw selection probabilities cannot be known for all primary units in the sample, but they are known for the networks that are encountered. The modified Hansen-Hurwitz estimator is

$$\hat{T}_{HH} = \frac{N}{n_1} \sum_{i=1}^{n_1} w_i \quad (9)$$

where w_i is the average of the y_i values in the network (A_i) that include the i^{th} unit of the initial sample of size n_1 :

$$w_i = \frac{1}{m_i} \sum_{j \in A_i} y_j, \quad (10)$$

and m_i is the number of primary units in network A_i . The sampling variance of this estimator can be calculated as

$$V(\hat{T}_{HH}) = \frac{N(N - n_1)}{n_1(n_1 - 1)} \sum_{i=1}^N (w_i - \mu)^2 \quad (11)$$

The relative efficiency of adaptive cluster sampling is calculated as

$$RE\left(\frac{ACS}{SRS}\right) = \frac{V(\hat{T}_{SRS})}{V(\hat{T}_{HH})} \quad \text{or} \quad \frac{V(\hat{T}_{SRS})}{V(\hat{T}_{HT})} \quad (12)$$

When $RE > 1$, the adaptive cluster sample is more efficient than the simple random sample (the design provides more accurate estimates of the population total at an equal sample size).

The variance of the SRS estimator in equation 12 was calculated given a sample size equal to the expected final sample size ($E(n^*)$) for the ACS design:

$$E(n^*) = \sum_{i=1}^N \pi_i, \quad (13)$$

where π_i is the probability of including unit i in the sample:

$$\pi_i = 1 - \left[\frac{\binom{N - m_i - a_i}{n_1}}{\binom{N}{n_1}} \right], \quad (14)$$

m_i is the number of units in the network to which unit i belongs, and a_i is the number of units in the network of which unit i is an edge unit. Note that both π_i and $E(n^*)$ can be determined only if the y -values and spatial locations of all basic sampling units are known.

Procedures

The starting point for this study was point distributions for six populations of rare, endangered plant species, with a wide range of abundances located in sites with a wide range of areas. These distributions were imported into a Geographical Information System (GIS) environment, where a variety of sampling scenarios based on changes in four design parameters were systematically applied to them. With each of these scenarios a specific set of GIS procedures (Appendix A) were used to extract information on universe attributes required for input into a set of specially designed R functions (Appendix B) for computing sampling variances using the two ACS variance estimators under different design conditions (R Development Core Team, 2004). These sampling

variances were then expressed as ratios relative to $V(\hat{T}_{SRS})$ in order to provide comparisons across all six populations.

Of the six populations utilized here, four populations were of Western Lily (*Lilium occidentale*) and two were of Kneeland prairie penny-cress (*Noccaea fendleri* ssp *californicum*). *Lilium occidentale* is an herbaceous perennial flowering plant and occurs in bogs or coastal scrub in a narrow band from Eureka, California to Coos Bay, Oregon. It is known to occur at 28 sites, only four with as many as 1000 individuals, three with 100 to 300 individuals and 21 with 100 or fewer. *Noccaea fendleri* ssp *californicum* only occurs within serpentine outcrops on Kneeland prairie (Humboldt Co, CA), numbering from 23 to 9,000 individuals in a given outcrop. The six populations employed in this study are summarized in Table 1. The X, Y coordinates for all individuals in the populations are accurate to at least 0.5 meters and were supplied by Dave Imper of the USFWS, collected as part of each species' management plan (U.S. Fish and Wildlife Service 1998, 2003). Site areas are approximate because the method used to overlay grids of varying scales sometimes resulted in total areas slightly larger than those stated.

Table 1. Summary information for case study populations. Population label, species name, site area, total abundance, average nearest neighbor (NN) distance and the standard deviation (SD) of nearest neighbor distance.

Label	Species	Area (m ²)	T	Avg NN (m)	NN SD (m)
1	<i>Lilium occidentale</i>	15525	74	3.36	7.53
2	<i>Lilium occidentale</i>	9600	80	1.98	2.60
3	<i>Noccaea fendleri</i> ssp <i>californicum</i>	9100	116	1.43	2.11
4	<i>Noccaea fendleri</i> ssp <i>californicum</i>	9100	189	1.03	1.87
5	<i>Lilium occidentale</i>	6000	625	0.09	0.41
6	<i>Lilium occidentale</i>	220400	1997	0.67	2.88

The point data for each population were imported into ArcGIS 9.1 (ESRI 2005) where grid-based sampling universes were simulated comprised of square primary units of lengths ranging from 1 m to 15 m. Prior to grid placement, a constant value of 0.01 was added to the X, Y coordinates of the individuals in all populations (Appendix A, step 3). As long as both this constant and the precision of the distance between gridlines were less than the precision of the point data, this technique effectively avoided the placement of a gridline directly upon a point. It should be noted that the effect of grid placement was not examined in this study, but is an issue that merits further investigation, as it has been shown to significantly affect the area of occupancy measures in grid-based settings such as this (White 2004).

Through further steps given in Appendix A, information required by functions written specifically for this study was exported and analyzed using computer programs written in R (R Development Core Team, 2004). Many current statistical analysis programs are not equipped to perform the necessary calculations to provide HT variances. Those that have the capabilities, such as R, don't directly provide a tool for it, nor was one available on the web. The HT Variance Calculator designed for this study makes use of GIS extracted data as well as sample data and is fairly easy to use. This program and all other R functions written and used for this study are presented and explained in Appendix B.

Four factors were considered in analyzing the performance of ACS relative to SRS. These factors were: a) nine spatial scales ranging from 1 m to 15 m, b) ten initial sampling fractions ranging from 0.01 to 0.50, c) two estimators (HT & HH), and d) two

conditions to include additional units in a sample ($C_1: \{i: y_i > 0\}$, $C_2: \{i: y_i > 1\}$). The relative efficiency of an ACS estimator given an initial sample size of n_l compared to a SRS estimator given a sample size of $E(n^*)$ was calculated for all possible combinations of these factors. While these comparisons were made at equal sample sizes, this does not necessarily imply equal sampling efforts, and no attempt was made in this study to correct for differences in distance traveled under SRS and ACS designs.

RESULTS

For all the populations examined in this study there were multiple conditions under which one or both of the ACS estimators were relatively more efficient than the SRS estimator given an equal final sample size. The Hansen-Hurwitz estimator performed uniformly worse than the Horvitz-Thompson estimator and rarely better than the SRS estimator, so only the performance of the HT estimator is presented here. Complete tables of the relative efficiencies of both the HT and HH estimators under all conditions are presented in Appendix D.

For ease of interpretation, contour plots are used to represent the relative efficiencies under the various sampling conditions. The leftmost contour line represents a relative efficiency of one ($RE=1$), so the combinations of sample unit edge lengths and sampling fractions that fall within the area above and to the right of this contour line represent conditions under which ACS would perform better than SRS. In many cases the increase in RE across the plot is quite large, and additional contour lines have been added to indicate trends. Because efficiency at smaller sampling fractions is usually more desirable, a vertical tangent to the $RE=1$ contour line will indicate optimal combinations of sample unit lengths and initial sampling fractions if they exist.

Population 1, with highly variable nearest neighbor distances (Table 1), roughly exhibits two optimal sample unit lengths; one at 4 meters and another at 7.5 meters (Fig. 2). This may be an indication of clustering on multiple scales. Population 2 exhibits an optimal sample unit length of 3 meters (Fig. 3). Population 6, with the lowest density of

points per area, had the most sets of conditions under which ACS outperformed SRS (Fig. 4). The flat appearance of the contour lines indicates that there was no sample unit length that could be considered “optimal” with respect to the other lengths. On the other hand, Population 5, with the highest density of points per area, displays an obvious optimal sample unit length of approximately 2 meters (Fig. 5). As units increase in length, the performance of ACS at sampling this population decreases steadily. Incidentally, Population 5 also had the smallest and least variable nearest neighbor distances (Table 1). Populations 3 and 4, also with smaller nearest neighbor distances, displayed a similar, if not quite as obvious trend of better performance at smaller sample unit lengths (Figs. 6 & 7). In general, ACS appears to perform well at sample unit lengths of 2 to 5 meters for all populations.

In general, the ratio of the expected final sample size ($E(n^*)$) with an ACS design relative to the initial sample size (n_I) increased as the size of the basic sampling unit increased and decreased as the initial sampling fraction increased. The pattern exhibited by Population 6 (Fig. 8) is typical of the patterns exhibited by the other populations in this study with regards to the relative magnitude of the expected final sample size. There were no clear relationships between the expected final sample sizes and the relative efficiency values for any of the populations.

Some interesting trends were also observed across increasing sampling fractions. The efficiency of both ACS estimators increased as initial sampling fraction increased, but the RE of the HH estimator showed modest increases at best (Appendix D). On the other hand, the HT estimator exhibited extremely dramatic increases in efficiency with only modest increases in sampling fraction. For example, the efficiency of the HT

estimator at sampling Population 5 with a sample unit length of 2 meters and condition C_1 increased 50 fold as initial sampling fraction increased from 0.025 to 0.10 (Appendix D, Table 18). This is an extreme example however, as Population 5 contains a “hyper-network”, a particularly large network that contains a large fraction of the population’s individuals. As the initial sampling fraction increases so does the probability of including this hyper-network, resulting in very low sampling variances. In addition to this extreme example, the HT estimator showed similar trends of taking better advantage of larger sample fractions than the SRS estimator in the other populations as well.

When the condition C was made more restrictive, even though it was by the smallest degree possible, significant changes in the relative efficiency of the ACS HT estimator can be seen in all six populations. For all populations, condition C_2 required a higher initial sampling fraction for ACS to match the efficiency of SRS. It is important to note, however, that the more restrictive condition C_2 resulted in final sample sizes being 20% smaller than under the less restrictive condition C_1 . The SRS sampling variances used in determining the REs under condition C_2 were also calculated given these smaller sample sizes, but did not exhibit as much of a decrease in precision due to the fact that with a more restrictive condition the network sizes in a population will decrease on average and the proportion of edge units to network units will increase. This causes the proportion of the sample that is explicitly used by the HT estimator to go down, bringing precision down as well. Despite the lower performance of the ACS HT estimator under the condition C_2 , the contour plots seen in figures 2-7 still indicate similar optimal sample unit sizes.

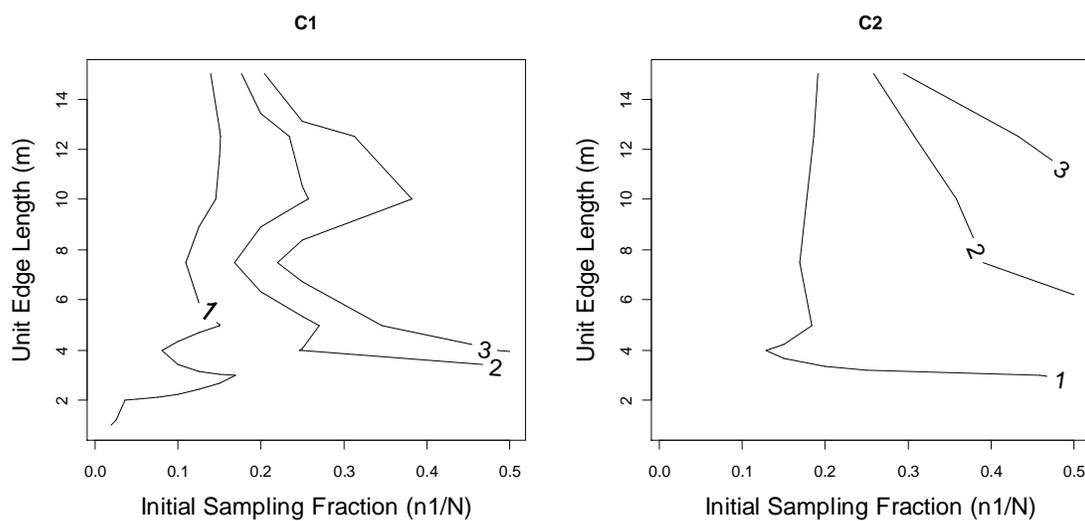


Figure 2. Contour plots representing the efficiency of the HT estimator at sampling Population 1 ($T=74$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

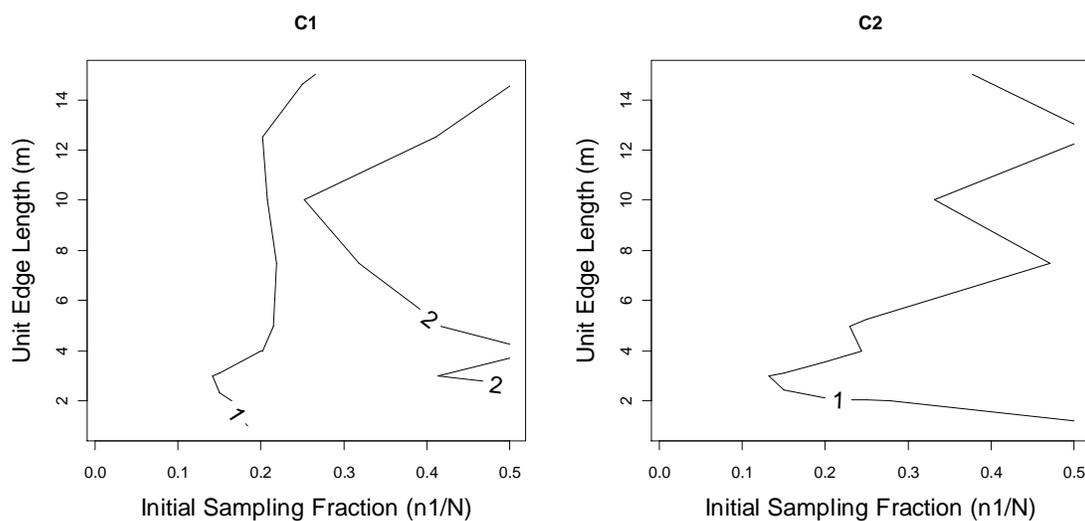


Figure 3. Contour plots representing the efficiency of the HT estimator at sampling Population 2 ($T=80$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

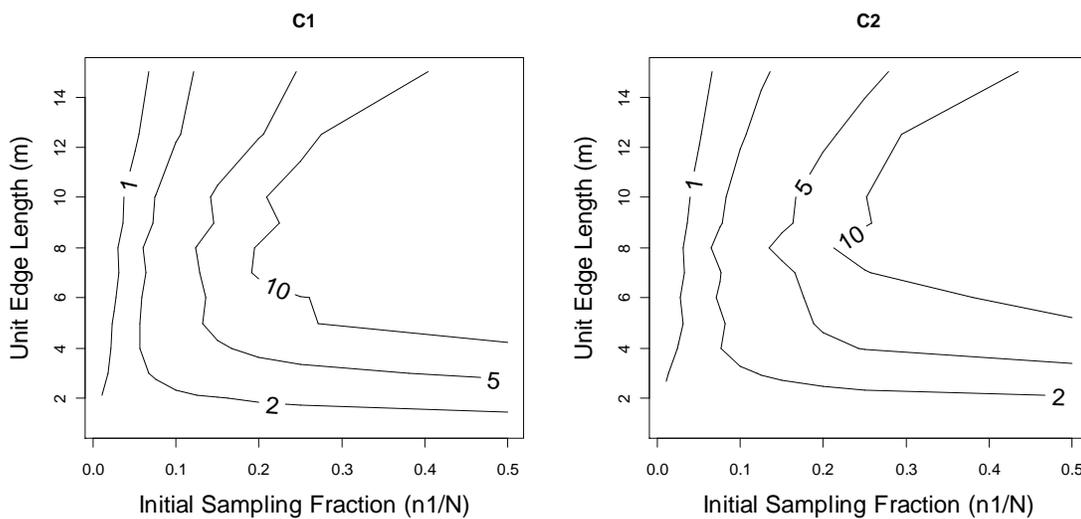


Figure 4. Contour plots representing the efficiency of the HT estimator at sampling Population 6 ($T=1997$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

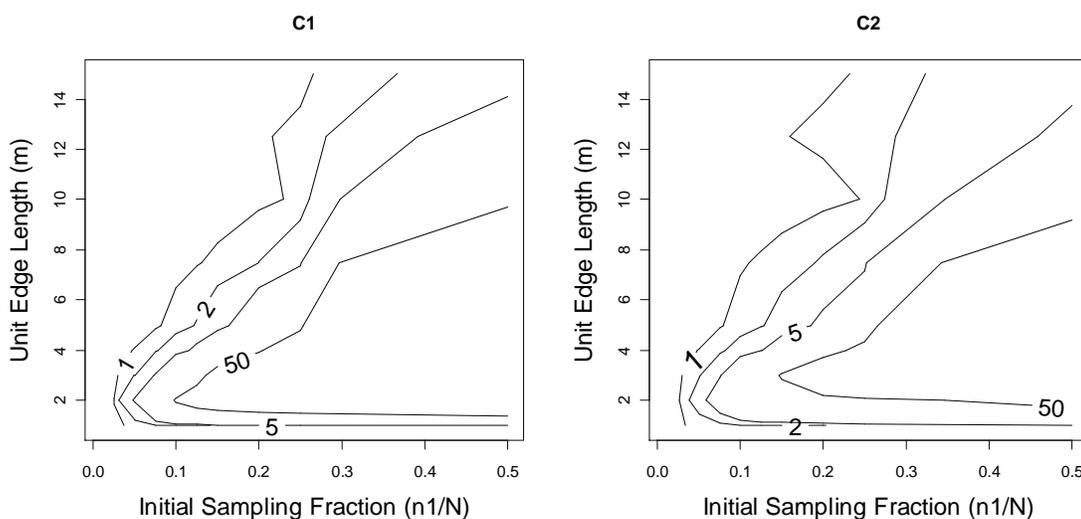


Figure 5. Contour plots representing the efficiency of the HT estimator at sampling Population 5 ($T=625$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

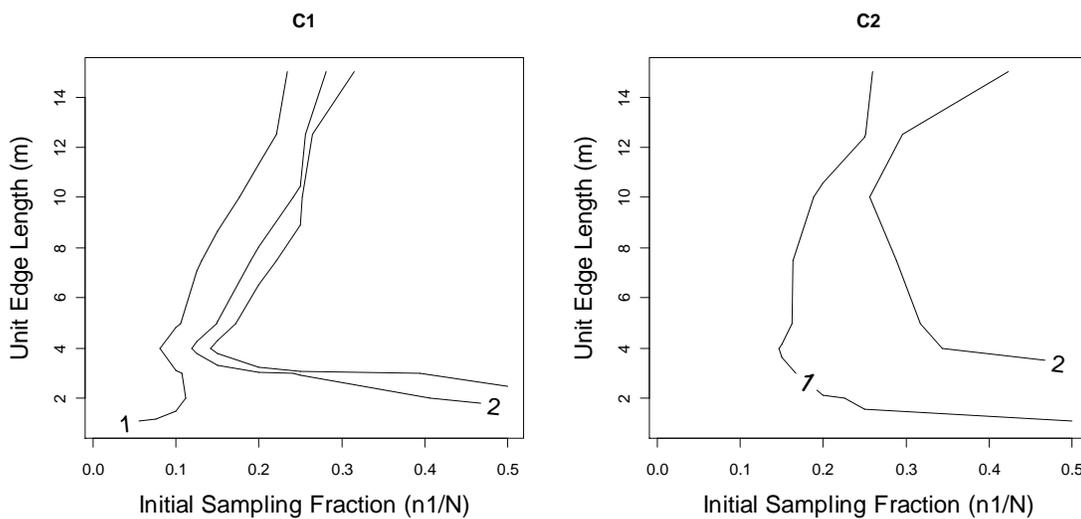


Figure 6. Contour plots representing the efficiency of the HT estimator at sampling Population 3 ($T=116$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

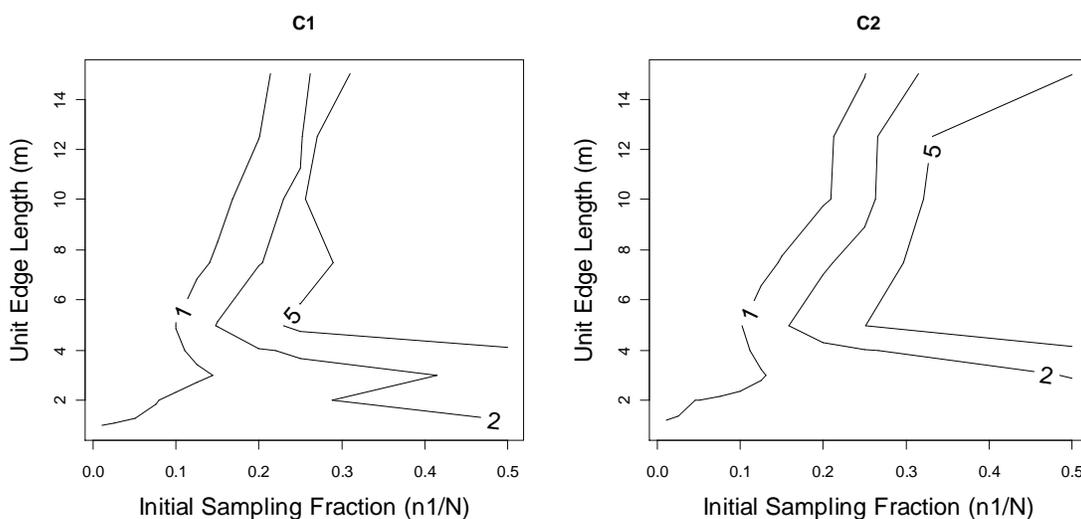


Figure 7. Contour plots representing the efficiency of the HT estimator at sampling Population 4 ($T=189$) relative to the SRS estimator given two conditions to include additional units in a sample: $C_1 \{i: y_i > 0\}$ and $C_2 \{i: y_i > 1\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

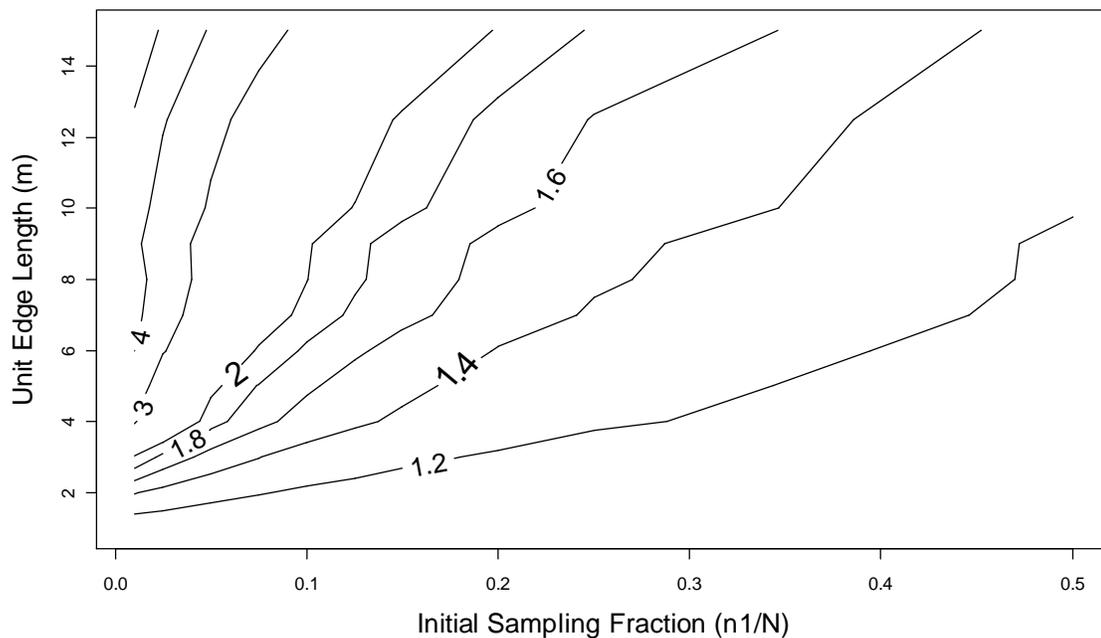


Figure 8. Contour plot representing the ratio of the expected final sample size ($E(n^*)$) to the initial sample size (n_I) for an ACS design applied to Population 6 ($T=1997$) with a condition to include additional units in a sample of $C_I \{i: y_i > 0\}$. Initial sampling fractions range from 0.01 to 0.50 and sample unit edge lengths range from 1 m to 15 m.

DISCUSSION

The efficiency of ACS depends on a number of factors. According to earlier work, higher efficiencies result when the final sample size is only slightly larger than the initial sample size, as well as when the within-network variance (as opposed to between-network variance) is a large proportion of the total variance (Smith et al. 1995, Thompson & Seber 1996, Brown 2003, Christman 1997). This is difficult to achieve, though, because these two circumstances are usually mutually exclusive. What's more, these suggestions are mainly based on the behavior of the Hansen-Hurwitz estimator, and may not necessarily hold true for the Horvitz-Thompson estimator. Smith et al. (2004) argued that the efficiency of an adaptive cluster sample is a function of the interaction between the within-network variance and final sample size and ultimately depends upon the spatial distribution of the target population. This study shows that a wide variety of populations, at the appropriate scale and given the appropriate condition for adaptive expansion of the sample, can be sampled more efficiently with an adaptive cluster sample design. The key is to be able to determine what scale, initial sampling fraction, and condition to adaptively add units to the sample is suitable.

The results of this study also indicate that there should be no question that the HT estimator is superior to the HH estimator for use in an ACS setting. Although the HH estimator uses the adaptively added units to adjust the values of the initially sampled units to network means, it does not explicitly incorporate any of these additional unit values directly into the estimate. With calculators such as that presented in Appendix B,

ease of use should no longer be an issue regarding the application of the HT estimator to an ACS design.

The choice of the condition, C , to include additional units in a sample is difficult. A less restrictive condition will result in a higher proportion of the population to be sampled, but can result in a final sample size that is much larger than the initial sample and more costly to implement. At the same time, it can also result in the within-network variance being a high fraction of the total variance, which will increase the efficiency of the adaptive cluster sample and provide highly precise estimates. In a simulation study, Brown (2003) found that a less restrictive condition did indeed result in high relative efficiencies for some populations, but only those with low total abundances. A highly restrictive condition will result in lower final sample sizes and less empty edge units being sampled, but for some populations it may result in little additional information being added to the sample and the full benefit of an adaptive cluster design may be lost. However, for populations that seem not to exhibit the appropriate level of geographical rarity for the practical implementation of ACS, the choice of a restrictive condition can result in geographically rare networks that will add information and precision to the sample without the danger of an exorbitant final sample size. This study showed that even the smallest change in the condition to adaptively add units to the sample can have strong effects on both the efficiency of ACS and the final size of the sample. Ultimately the choice of all design parameters, but especially the choice of condition C , will have to be made in the context of time, resources, and other issues.

The reaction of the HT estimator to increasing sample size has implications for sample budgeting. If a small increase in the initial sample can double or triple the precision of the estimates it returns, larger sampling budgets may be much more justifiable. However, a close examination of the tables in Appendix D shows that the response of the HT estimator to increasing sample sizes can vary in magnitude quite a bit between grid cell sizes that are very similar. A decision regarding the initial sampling fraction should not be made without carefully considering the size of the basic sampling unit at the same time.

Very few studies have ever examined the effects of sample unit size on the performance of ACS. Simulation studies like those done by Christman (1997), and Brown (2003) examine a wide range of other design parameters but did not actually examine the effect of sample unit size, even though they acknowledge its likely importance. In Phillippi's (2005) application of ACS to some small plant populations, he used just two sample unit sizes, the larger of which was a multiple of the smaller. He found that while different final sample sizes resulted, there was no difference in precision. The results of this study, which examined a far greater range of sample unit sizes than any previous study, show that sample unit size can have as much if not more of an impact on the performance of ACS than other design parameters. The question of how to determine the optimal sample unit size is a difficult one, especially from limited pre-sample information. For the populations examined in this study, average nearest neighbor distances tended to be smaller than sample unit lengths indicated as optimal by the RE contour plots (Figs, 2-7). Until more research is done, a tentative guideline is to scale the

sample units so they are at least larger than the average nearest neighbor distance.

Unfortunately, obtaining a reliable estimate of average nearest neighbor distance may be just as much of a problem as estimating a population's total abundance.

The findings of this study reveal that ACS estimates of abundance have a sensitivity to design parameters, such as sample unit size, that has not yet received enough attention to be fully understood. Unfortunately, given the impact these design parameters can have on the efficiency of a sample, it will be very difficult to ever define a rule for constructing an optimal adaptive cluster sample because the effects of the design parameters on an ACS estimator's performance within a grid-based setting depends on the specific point distribution of individuals across the sample space, the knowledge of which is always incomplete prior to performing a sample. This raises the question of whether information from previous studies or preliminary surveys can be effectively used to optimize ACS parameters, and to what degree the procedures introduced here can be utilized to that aim. With further research and the development of software applications, future researchers could not only be able to compare a range of design parameters, but also a range of different designs at the same time. This may be done not only with fully mapped populations, but with simulated populations based upon population distribution characteristics derived from sample data as well. Monitoring activities especially have the potential to be improved through the use of these procedures. Researchers will be well positioned to make use of data from preceding years to compare a wide variety of design parameters. Sampling procedures could be adjusted and refined, potentially resulting in considerably reduced sampling costs with equal or even more precise estimates of

abundance. Through the innovative use of tools such as Geographic Information Systems, flexible sampling designs like ACS may no longer be seen as overly complicated or intimidating. Instead, the very flexibility that has at times made ACS difficult to implement effectively could now allow this design to be tailored to many diverse sampling situations, and the use of this and other such adaptive designs may expand.

REFERENCES

- Acharya B, Bhattarai G, Gier A, Stein A. 2000. Systematic adaptive cluster sampling for the assessment of rare tree species in Nepal. *Forest Ecology and Management* 137(2000):65-73.
- Borkowski J. 1999. Network inclusion probabilities and Horvitz-Thompson estimation for adaptive simple Latin square sampling. *Environmental and Ecological Statistics* 6:291-311.
- Brown JA, Manly BJB. 1998. Restricted adaptive cluster sampling. *Environmental and Ecological Statistics* 5:49-63.
- Brown JA. 2003. Designing an efficient adaptive cluster sample. *Environmental and Ecological Statistics* 10:95-105.
- Christman MC, Pontius JS. 2000. Bootstrap confidence intervals for adaptive cluster sampling. *Biometrics* 56(2):503-510.
- Christman MC. 1997. Efficiency of some sampling designs for spatially clustered populations. *Environmetrics* 8:145-166.
- Christman MC. 2000. A review of quadrat-based sampling of rare, geographically clustered populations. *Journal of Agricultural, Biological, and Environmental Statistics* 5(2):168-201.
- Dryver AL, Thompson SK. 2005. Improving unbiased estimators in adaptive cluster sampling. *Journal of the Royal Statistical Society B* 67(1):157-166.
- Goldberg NA, Heine JN, Brown JA. 2007. The application of adaptive cluster sampling for rare subtidal macroalgae. *Marine Biology* 151:1343-1348.
- Hanselman DH, Quinn TJ, Lunsford C, Heifetz J, Clausen D. 2003. Applications in adaptive cluster sampling of Gulf of Alaska rockfish. *Fisheries Bulletin* 101:501-513.
- Imper DK, Sawyer JO. 1990. 1989 Monitoring Report for the Western Lily. California Department of Fish and Game. 25 pages.
- Lo NCH, Griffith D, Hunter JR. 1997. Using a restricted adaptive cluster sampling to estimate pacific hake larval abundances. *California Cooperative Oceanic Fisheries Investigations Reports* 38:103-113.

- Magnussen S, Kurz W, Leckie DG, Paradine D. 2005. Adaptive cluster sampling for estimation of deforestation rates. *Eur J Forest Res* 124:207-220.
- Morrison LW, Smith DR, Young CC, Nichols DW. 2008. Evaluating sampling designs by computer simulation: a case study with the Missouri bladderpod. *Population Ecology* 50(4):417-425.
- Noon BR, Ishwar NM, Vasudevan K. Efficiency of adaptive cluster and random sampling in detecting terrestrial herpetofauna in a tropical rainforest. *Wildlife Society Bulletin* 34(1):59-68.
- Philippi T. 2005. Adaptive cluster sampling for estimation of abundances within local populations of low-abundance plants. *Ecology* 86(5): 1091-1100.
- R Development Core Team (2004) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna.
<http://www.R-project.org>
- Salehi MM. 1999. Rao-Blackwell versions of the Horvitz-Thompson and Hansen Hurwitz in adaptive cluster sampling. *Environmental and Ecological Statistics* 6:183-195.
- Salehi MM. 2003. Comparison between Hansen-Hurwitz and Horvitz-Thompson estimators for adaptive cluster sampling. *Environmental and Ecological Statistics* 10:115-127.
- Skibo KM, Schwarz CJ, Peterman RM. 2008. Evaluation of sampling designs for Red Sea Urchins *Strongylocentrotus franciscanus* in British Columbia. *North American Journal of Fisheries Management* 28:219-230.
- Smith DR, Conroy MJ, Brakhage DH. 1995. Efficiency of adaptive cluster sampling for estimating density of wintering waterfowl. *Biometrics* 51(2):777-788.
- Smith DR, Vilella RF, Lemarie DP. 2003. Application of adaptive cluster sampling to low-density populations of freshwater mussels. *Environmental and Ecological Statistics* 10:7-15.
- Smith DR, Brown JA, Lo NCH. 2004. Application of adaptive sampling to biological populations. In: Thompson WL, editor. *Sampling rare or elusive species: concepts, designs, and techniques for estimating population parameters*. Washington (DC): Island Press. p. 75-122.

- Sullivan WP, Morrison BJ, Beamish FWH. 2008. Adaptive cluster sampling: estimating density of spatially autocorrelated larvae of the sea lamprey with improved precision. *Journal of Great Lakes Research* 34:86-97.
- Talvitie M, Leino O, Holopainen M. 2006. Inventory of sparse forest populations using adaptive cluster sampling. *Silva Fennica* 40(1):101-108.
- Thompson SK. 1990. Adaptive cluster sampling. *Journal of the American Statistical Association* 85(412):1050-1059.
- Thompson SK. 1991a. Adaptive cluster sampling: designs with primary and secondary units. *Biometrics* 47(3):1103-1115.
- Thompson SK. 1991b. Stratified adaptive cluster sampling. *Biometrika* 78(2):389-397.
- Thompson SK. 1996. Adaptive cluster sampling based on order statistics. *Environmetrics* 7:123-133.
- Thompson SK, Seber GAF. 1996. *Adaptive Sampling*. New York (NY): Wiley.
- Tsiatis AA, Mehta C. 2003. On the inefficiency of the adaptive design for monitoring clinical trials. *Biometrika* 90(2):367-378.
- U.S. Fish and Wildlife Service (1998a) Recovery Plan for the Endangered Western lily (*Lilium occidentale*). Portland, Oregon. 82 pp.
- U.S. Fish and Wildlife Service (2003) Recovery Plan for the Kneeland Prairie Penny-cress (*Thlaspi californicum*). Portland, Oregon. Vii + 42 pp.
- White JW .2004. Range size, error rates, and the geometry of rare species distributions. In: Brooks MB, Carothers SK, LaBanca T, editors. *The Ecology and Management of Rare Plants of Northwestern California: Proceedings from a 2002 Symposium of the North Coast Chapter of the California Native Plant Society*. California Native Plant Society, Sacramento, CA. p. 11-20.
- Wiegand T, Gunatilleke S, Gunatilleke N, Okuda T. 2007. Analyzing the spatial structure of a Sri Lankan tree species with multiple scales of clustering. *Ecology* 88(12):3088-3102.

APPENDIX A

GIS procedures using ArcGIS 9.1

1. Download and install HawthTools (Beyer 2004) available free at <http://www.spatial ecology.com/htools>
2. Make sure point X,Y coordinates saved as a dbase file (.dbf) with separate, labeled columns for the X and Y values. Import into ArcGIS using <Add Data>
3. Data Management Tools -> Fields -> Calculate Fields
Select the coordinate dbase file from step 2 as the Input Table.
Select the X-coordinate column label as the Field Name.
Write expression to add a value less than the data's level of precision.
Repeat for Y-coordinate.
4. Tools -> Add XY Data
Creates a point distribution using the XY data.
5. HawthTools -> Sampling Tools -> Create Vector Grid
Set the extent to the dimensions of the study area.
Set the spacing between lines to the desired sample unit length.
Set the output as a polygon layer.
Output is a grid polygon layer.
6. HawthTools-> Analysis Tools -> Count Points in Polygons
Adds PNTPOLYCNT field to the attribute table of a grid polygon layer.
7. ArcToolbox -> Conversion Tools -> To Raster -> Feature to Raster
The input is a grid polygon layer.
Set the field as PNTPOLYCNT.
Output is a density raster layer.
Open attribute table -> Options -> Export
The fields Value and Count correspond to vectors "yi" and "count" in the Simple Random Sampling Variance Calculator (Appendix B).
8. ArcToolbox -> Spatial Analyst Tools -> Reclassify
For values in a density raster not satisfying C, reclassify to "no data"
For all other values reclassify to 1
Output is a network raster layer.

9. ArcToolbox -> Conversion Tools -> from Raster -> Raster to Polygon
 The input is a network raster.
 Make sure “simplify polygons” is deselected.
 Output is a network polygon layer.
10. HawthTools-> Table Tools -> Add AREA/PERIMETER Fields to Table
 The input is a network polygon layer.
 Convert Area by the decimal equivalent of $1/(\text{sample unit length}^2)$
 Convert Perimeter by the decimal equivalent of $1/(\text{sample unit length})$
 Open attribute table -> Options -> Export
 The fields AREA and PERIMETER correspond to vectors “mk” and “ak”
 in the Final Sample Size Calculator (Appendix B).
- * For the condition $C \{i: y_i > 0\}$, proceed to step 10 and then stop. For all other more restrictive conditions, skip to step 11.
11. HawthTools-> Analysis Tools -> Count Points in Polygons
 Adds PNTPOLYCNT field to the attribute table of a network polygon layer.
 Open attribute table -> Options -> Export
 The fields AREA and PNTPOLYCNT correspond to vectors “mk” and
 “yk” in the Horvitz-Thompson Variance Calculator and the
 Hansen-Hurwitz Variance Calculator (Appendix B).
12. ArcToolbox -> Analysis Tools -> Extract -> Select
 Write SQL expression to select all values that fail to satisfy the condition C .
 Output is a polygon layer of occupied units that lead no further sampling.
13. ArcToolbox -> Analysis Tools -> Overlay -> Union
 Inputs are a polygon layer of occupied units that lead no further sampling and a
 network polygon layer.
 Output is a polygon layer of unique networks.
14. HawthTools-> Analysis Tools -> Count Points in Polygons
 Adds PNTPOLYCNT field to the attribute table of a polygon layer of unique
 networks.
 Open attribute table -> Options -> Export
 The fields AREA and PNTPOLYCNT correspond to vectors “mk” and
 “yk” in the Horvitz-Thompson Variance Calculator and the
 Hansen-Hurwitz Variance Calculator (Appendix B).

APPENDIX B

R Functions

The Horvitz-Thompson Variance Calculator

#This function calculates the sampling variance of the Horvitz-Thompson estimator of #total abundance in an Adaptive Cluster Sampling design.

#Arguments:

```
# data: list created by using the read.dbf function on the exported attribute
# table of the network polygon layer from Appendix A
# N: total units in population
# sampfrac: vector of sampling fractions (n/N) to be selected from N
# mk: vector of total units that comprise each network k in the population
# yk: vector of total y-values in each network k
```

```
var_tau_ht<-function(N,sampfrac,data)
```

```
{
  mk<-data[["AREA"]]
  yk<-data[["PNTPOLYCNT"]]
  varht<-function(N,n,mk,yk)
  {
    ck<-exp(lfactorial(N-mk)-lfactorial(N-mk-n)-lfactorial(N)+lfactorial(N-n))
    mkh<-colSums(combn(mk,2))
    ckh<-exp(lfactorial(N-mkh)-lfactorial(N-mkh-n)-lfactorial(N)+lfactorial(N-n))
    alphak<-1-ck
    cknh<-colSums(combn(ck,2))
    alphakh<-1-(cknh-ckh)
    yxy<-combn(yk,2)
    yprod<-(yxy[1,])*(yxy[2,])
    alphaxalpha<-combn(alphak,2)
    alphaprod<-(alphaxalpha[1,])*(alphaxalpha[2,])
    alphasqr<-alphak*alphak
    ysqr<-yk*yk
    vtermkh<-yprod*(alphakh-alphaprod)/alphaprod
    vtermkk<-ysqr*(alphak-alphasqr)/alphasqr
    varterms<-c(vtermkh,vtermkk)
    var<-(sum(varterms))
    var
  }
  sampsize<-(N*sampfrac)
  for(n in sampsize)print(varht(N,n,mk,yk))
}
```

The Hansen-Hurwitz Variance Calculator

#This function calculates the variance of the Hansen-Hurwitz estimator of total abundance in an Adaptive
#Cluster Sampling design.

#Arguments:

data: list created by using the read.dbf function on the exported attribute table of the
network polygon layer from Appendix A
N: total units in population
sampfrac: vector of sampling fractions (n/N) to be selected from N
mk: vector of total units that comprise each network k in the population
yk: vector of total y-values in each network k

```
var_tau_hh<-function(N,sampfrac,data)
{
  mk<-data[["AREA"]]
  yk<-data[["PNTPOLYCNT"]]
  var_hh<-function(N,n,mk,yk)
  {
    wi<-yk/mk
    mu<-(sum(yk))/N
    ss<-sum(((wi-mu)^2)*mk)
    emptyss<-((0-mu)^2)*(N-(sum(mk)))
    c<-(N*(N-n))/(n*(N-1))
    var<-c*(ss+emptyss)
    var
  }
  sampsize<-(N*sampfrac)
  for(n in sampsize)print(var_hh(N,n,mk,yk))
}
```

The Final Sample Size Calculator

#This function calculates the expected final sample size under an Adaptive Cluster Sampling design.

#Arguments:

```
# data: list created by using the read.dbf function on the exported attribute table of the
# network polygon layer from Appendix A
# N: total units in population
# sampfrac: vector of sampling fractions (n/N) to be selected from N
# mk: vector of total units that comprise each network k in the population
# ak: vector of total edge units for each network k
```

```
exp_fin_n<-function(N,sampfrac,data)
```

```
{
  mk<-data[["AREA"]]
  ak<-data[["PERIMETER"]]
  nstar<-function(N,n,mk,ak)
  {
    edgepi<-(ak)*(1-(exp(lfactorial(N-1-mk)-lfactorial(N-1-mk-n)-lfactorial(N)+lfactorial(N-n))))
    netpi<-(mk)*(1-(exp(lfactorial(N-mk)-lfactorial(N-mk-n)-lfactorial(N)+lfactorial(N-n))))
    netandedge<-c(mk,ak)
    others<-(N-sum(netandedge))
    otherpi<-(others)*(1-(exp(lfactorial(N-1)-lfactorial(N-1-n)-lfactorial(N)+lfactorial(N-n))))
    nfinal<-sum(edgepi,netpi,otherpi)
    nfinal
  }
  sampsize<-(N*sampfrac)
  for(n in sampsize)print(nstar(N,n,mk,ak))
}
```

The Simple Random Sampling Variance Calculator

#This function calculates the variance of the Simple Random Sample estimator of total abundance in an #Adaptive Cluster Sampling design.

#Arguments:

data: list created by using the read.dbf function on the exported attribute table of the
density raster layer from Appendix A
N: total units in sample universe
sampsize: vector of sampling sizes from the output of the Final Sample Size calculator
yi: value vector for the y_i values in a population
count: frequency vector for the y_i values in a population

```
>var_t_srs<-function(sampsize,data)
{
  yi<-data[["Value"]]
  count<-data[["Count"]]
  srsvar<-function(n,yi,count)
  {
    N<-sum(count)
    total<-sum(yi*count)
    mu<-(total/N)
    popvar<-((sum(((yi-mu)^2)*count))/(N-1))
    srsvar<-(N-n)*(N/n)*popvar
    srsvar
  }
  for(n in sampsize)print(srsvar(n,yi,count))
}
```

APPENDIX C

Point Distribution Maps

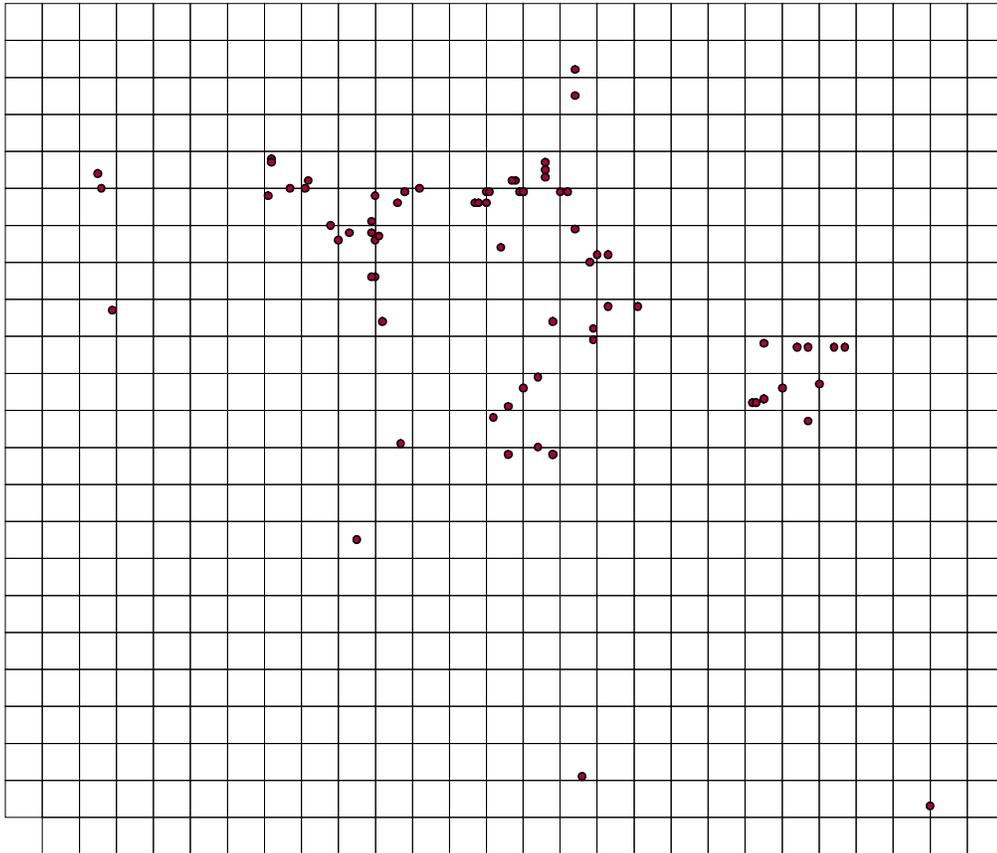


Figure 9. Point distribution of Population 1 (74 individuals in a 15,525 m² area). A grid with unit lengths of 5 meters has been overlaid to provide scale.

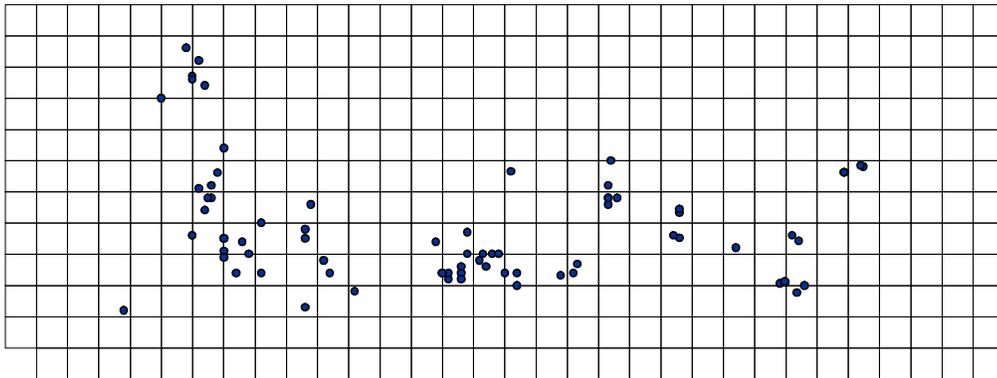


Figure 10. Point distribution of Population 2 (80 individuals in a 9,600 m² area). A grid with unit lengths of 5 meters has been overlaid to provide scale.

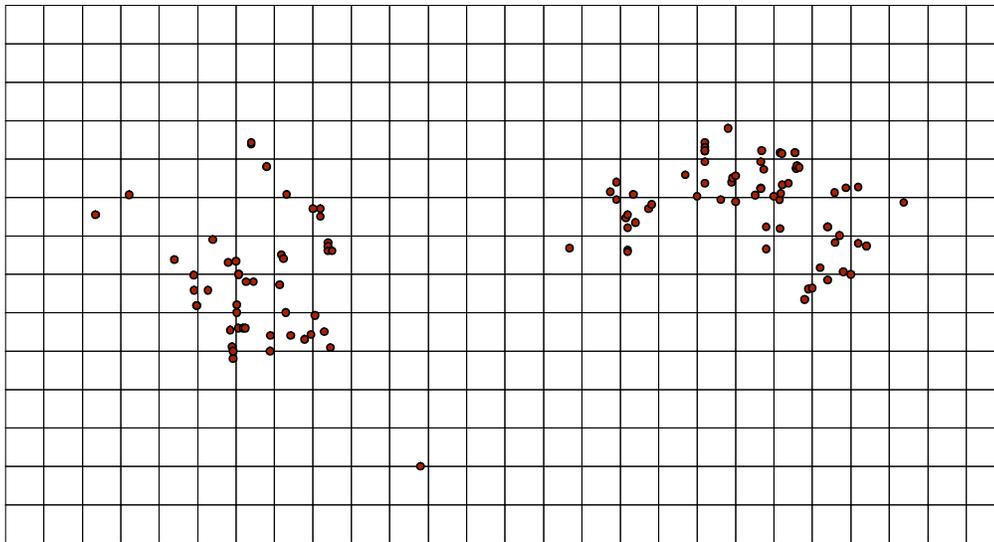


Figure 11. Point distribution of Population 3 (116 individuals in a 9,100 m² area). A grid with unit lengths of 5 meters has been overlaid to provide scale.

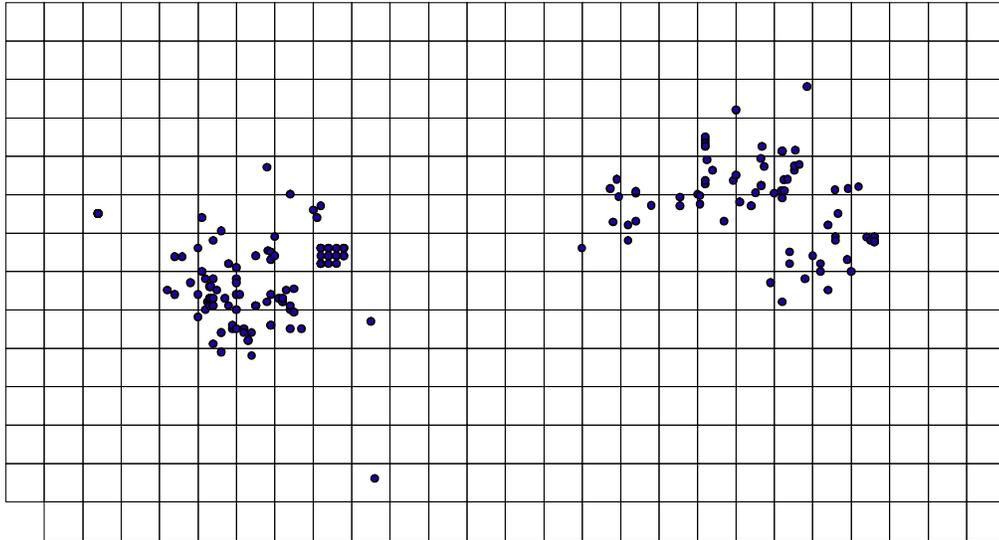


Figure 12. Point distribution of Population 4 (189 individuals in a 9,100 m² area). A grid with unit lengths of 5 meters has been overlaid to provide scale.

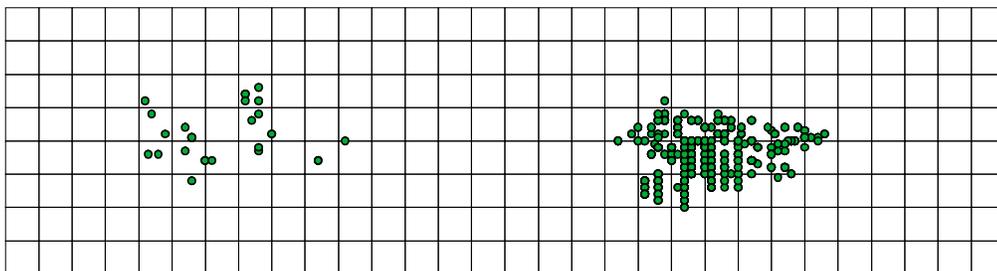


Figure 13. Point distribution of Population 5 (625 individuals in a 6000 m² area). A grid with unit lengths of 5 meters has been overlaid to provide scale.

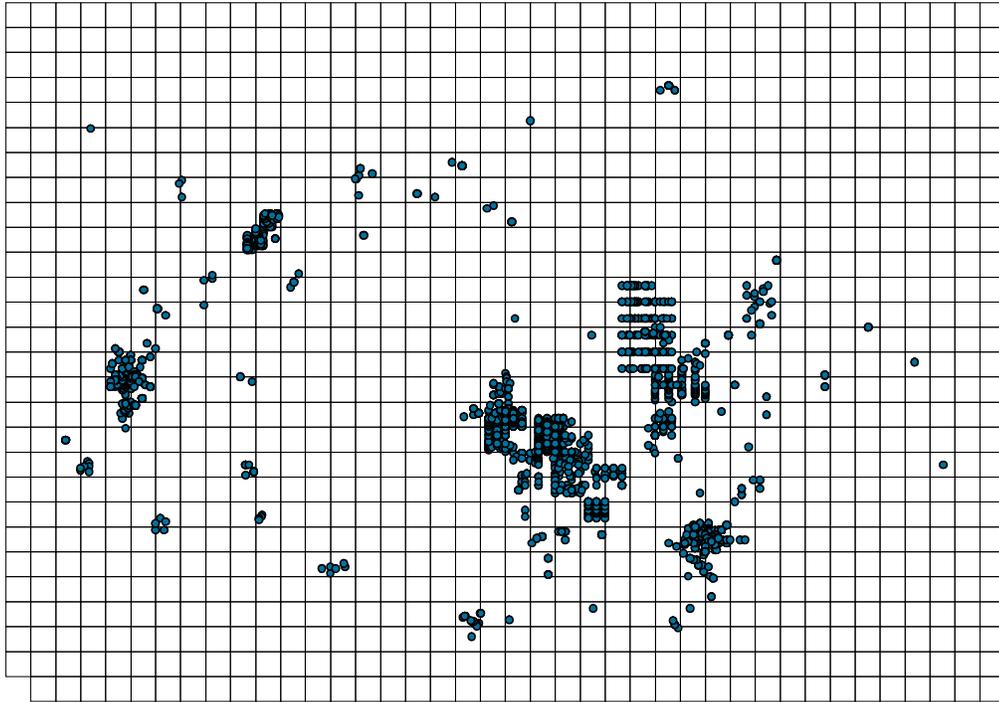


Figure 14. Point distribution of Population 6 (1,997 individuals in a 220,400 m² area). A grid with unit lengths of 15 meters has been overlaid to provide scale.

APPENDIX D

Relative Efficiency Tables

Population 1

Table 2. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 1 ($T=74$, Area= 15525 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_1 / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.99	0.78	0.61	0.48	0.24	0.20	0.24	0.15
0.025	1.00	0.99	0.80	0.68	0.52	0.30	0.24	0.27	0.17
0.050	1.01	1.01	0.83	0.81	0.59	0.43	0.33	0.35	0.23
0.075	1.02	1.02	0.87	0.96	0.67	0.62	0.46	0.45	0.33
0.100	1.03	1.03	0.90	1.12	0.76	0.88	0.62	0.58	0.49
0.125	1.03	1.05	0.94	1.29	0.87	1.22	0.82	0.76	0.75
0.150	1.04	1.06	0.97	1.45	1.00	1.64	1.05	0.98	1.17
0.200	1.06	1.09	1.04	1.76	1.32	2.61	1.52	1.54	2.75
0.250	1.08	1.13	1.11	2.02	1.74	3.57	1.95	2.20	5.50
0.500	1.21	1.36	1.44	3.06	5.03	7.63	3.94	5.41	13.37

Table 3. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 1 ($T=74$, Area= 15525 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_1 / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.98	0.95	0.91	0.72	0.54	0.24	0.25	0.22
0.025	1.00	0.98	0.95	0.92	0.74	0.57	0.27	0.27	0.24
0.050	1.00	0.98	0.95	0.94	0.78	0.63	0.34	0.33	0.28
0.075	1.00	0.98	0.95	0.96	0.82	0.69	0.43	0.40	0.34
0.100	1.00	0.98	0.96	0.98	0.86	0.77	0.54	0.49	0.42
0.125	1.00	0.98	0.96	1.00	0.90	0.84	0.67	0.60	0.52
0.150	1.00	0.98	0.96	1.02	0.94	0.93	0.82	0.74	0.66
0.200	1.00	0.98	0.97	1.06	1.03	1.12	1.15	1.10	1.08
0.250	1.00	0.98	0.97	1.10	1.12	1.33	1.48	1.53	1.78
0.500	1.00	0.98	1.01	1.36	1.53	2.51	2.67	3.54	8.63

Table 4. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 1 ($T=74$, Area= 15525 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.99	0.78	0.59	0.48	0.23	0.20	0.25	0.15
0.025	1.00	0.99	0.79	0.62	0.49	0.25	0.21	0.25	0.15
0.050	1.00	0.99	0.80	0.66	0.51	0.28	0.24	0.27	0.16
0.075	1.00	0.99	0.81	0.69	0.53	0.31	0.26	0.29	0.17
0.100	1.00	0.99	0.82	0.73	0.54	0.35	0.30	0.31	0.18
0.125	1.00	0.99	0.83	0.76	0.56	0.39	0.33	0.33	0.20
0.150	1.00	0.99	0.83	0.78	0.58	0.42	0.36	0.35	0.22
0.200	1.00	1.00	0.85	0.83	0.61	0.48	0.42	0.40	0.26
0.250	1.00	1.00	0.86	0.86	0.64	0.54	0.47	0.45	0.29
0.500	1.00	1.01	0.90	0.95	0.77	0.72	0.64	0.60	0.44

Table 5. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 1 ($T=74$, Area= 15525 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.99	0.96	0.92	0.73	0.55	0.24	0.26	0.23
0.025	1.00	0.99	0.96	0.93	0.74	0.56	0.25	0.26	0.23
0.050	1.00	0.99	0.96	0.93	0.75	0.58	0.27	0.27	0.24
0.075	1.00	0.99	0.96	0.93	0.76	0.59	0.30	0.29	0.25
0.100	1.00	0.99	0.96	0.93	0.77	0.61	0.32	0.31	0.26
0.125	1.00	0.99	0.96	0.94	0.78	0.62	0.35	0.33	0.27
0.150	1.00	0.99	0.96	0.94	0.79	0.64	0.38	0.35	0.28
0.200	1.00	0.99	0.96	0.94	0.81	0.67	0.43	0.39	0.31
0.250	1.00	0.99	0.96	0.95	0.82	0.69	0.48	0.44	0.35
0.500	1.00	0.99	0.96	0.96	0.87	0.79	0.66	0.62	0.49

Population 2

Table 6. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 2 ($T=80$, Area= 9600 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.97	0.91	0.72	0.57	0.56	0.37	0.13	0.15	0.15
0.025	0.98	0.91	0.75	0.59	0.58	0.39	0.14	0.17	0.16
0.050	0.98	0.93	0.80	0.64	0.62	0.43	0.17	0.20	0.18
0.075	0.98	0.94	0.85	0.69	0.66	0.48	0.21	0.26	0.22
0.100	0.99	0.96	0.90	0.75	0.71	0.53	0.27	0.35	0.26
0.125	0.99	0.97	0.96	0.81	0.76	0.60	0.36	0.47	0.33
0.150	0.99	0.99	1.02	0.87	0.82	0.68	0.47	0.62	0.41
0.200	1.00	1.02	1.15	1.00	0.95	0.89	0.88	0.98	0.64
0.250	1.01	1.06	1.31	1.13	1.11	1.18	1.66	1.34	0.94
0.500	1.06	1.28	2.37	1.84	2.44	4.18	31.78	2.37	1.92

Table 7. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 2 ($T=80$, Area= 9600 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.99	0.98	0.91	0.80	0.72	0.52	0.26	0.36	0.37
0.025	0.99	0.98	0.92	0.81	0.74	0.53	0.26	0.37	0.38
0.050	0.99	0.98	0.94	0.83	0.76	0.54	0.28	0.38	0.39
0.075	0.99	0.98	0.96	0.85	0.79	0.56	0.30	0.39	0.41
0.100	0.99	0.99	0.98	0.87	0.82	0.58	0.32	0.41	0.42
0.125	0.99	0.99	0.99	0.89	0.85	0.60	0.34	0.42	0.44
0.150	0.99	0.99	1.01	0.92	0.88	0.62	0.37	0.44	0.46
0.200	0.99	0.99	1.05	0.96	0.95	0.67	0.44	0.48	0.52
0.250	0.99	1.00	1.09	1.00	1.03	0.72	0.54	0.53	0.58
0.500	0.99	1.02	1.32	1.22	1.51	1.04	1.94	0.89	1.40

Table 8. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 2 ($T=80$, Area= 9600 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_1/N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.97	0.91	0.73	0.57	0.57	0.39	0.14	0.16	0.16
0.025	0.97	0.91	0.73	0.58	0.58	0.39	0.14	0.15	0.16
0.050	0.97	0.91	0.74	0.60	0.58	0.39	0.14	0.15	0.15
0.075	0.98	0.92	0.75	0.62	0.59	0.40	0.14	0.16	0.15
0.100	0.98	0.92	0.76	0.64	0.60	0.40	0.14	0.17	0.15
0.125	0.98	0.92	0.77	0.65	0.61	0.41	0.15	0.18	0.15
0.150	0.98	0.92	0.78	0.67	0.62	0.42	0.16	0.20	0.16
0.200	0.98	0.93	0.80	0.70	0.63	0.44	0.17	0.23	0.17
0.250	0.98	0.93	0.82	0.73	0.65	0.46	0.20	0.28	0.19
0.500	0.98	0.95	0.90	0.83	0.74	0.59	0.31	0.45	0.30

Table 9. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 2 ($T=80$, Area= 9600 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_1/N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.99	0.93	0.82	0.74	0.54	0.28	0.39	0.41
0.025	1.00	0.99	0.93	0.82	0.75	0.54	0.28	0.39	0.40
0.050	1.00	0.99	0.93	0.83	0.75	0.55	0.27	0.39	0.40
0.075	1.00	0.99	0.93	0.83	0.76	0.55	0.27	0.39	0.40
0.100	1.00	0.99	0.94	0.84	0.77	0.56	0.27	0.39	0.39
0.125	1.00	0.99	0.94	0.84	0.77	0.56	0.27	0.39	0.39
0.150	1.00	0.99	0.94	0.85	0.78	0.57	0.27	0.40	0.39
0.200	1.00	0.99	0.95	0.86	0.79	0.58	0.28	0.40	0.39
0.250	1.00	0.99	0.95	0.86	0.80	0.59	0.29	0.41	0.39
0.500	1.00	0.99	0.97	0.90	0.85	0.65	0.35	0.48	0.43

Population 3

Table 10. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 3 ($T=116$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.76	0.50	0.28	0.23	0.32	0.28	0.27	0.46
0.025	1.00	0.79	0.56	0.36	0.28	0.36	0.30	0.29	0.47
0.050	1.01	0.85	0.67	0.56	0.41	0.44	0.34	0.32	0.50
0.075	1.02	0.90	0.80	0.89	0.61	0.55	0.40	0.35	0.53
0.100	1.03	0.97	0.95	1.41	0.91	0.71	0.48	0.40	0.57
0.125	1.04	1.04	1.12	2.22	1.36	0.93	0.59	0.46	0.61
0.150	1.05	1.11	1.30	3.41	2.02	1.23	0.74	0.55	0.67
0.200	1.07	1.26	1.68	7.16	4.27	2.21	1.23	0.80	0.83
0.250	1.10	1.43	2.06	12.08	8.04	4.09	2.17	1.26	1.08
0.500	1.26	2.34	3.69	24.39	27.46	73.84	82.47	30.86	8.55

Table 11. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 3 ($T=116$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.99	0.94	0.78	0.60	0.36	0.44	0.33	0.34	0.52
0.025	0.99	0.94	0.80	0.63	0.39	0.47	0.35	0.36	0.54
0.050	0.99	0.95	0.84	0.69	0.46	0.54	0.40	0.39	0.56
0.075	0.99	0.95	0.87	0.76	0.55	0.61	0.46	0.42	0.59
0.100	0.99	0.96	0.90	0.84	0.65	0.70	0.53	0.46	0.63
0.125	0.99	0.97	0.94	0.92	0.78	0.81	0.62	0.51	0.66
0.150	0.99	0.98	0.98	1.01	0.92	0.92	0.74	0.57	0.71
0.200	0.99	0.99	1.05	1.22	1.25	1.22	1.08	0.73	0.81
0.250	0.99	1.01	1.12	1.46	1.63	1.58	1.63	0.97	0.94
0.500	0.99	1.09	1.45	2.89	3.02	4.31	18.93	6.68	2.47

Table 12. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 3 ($T=116$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_1/N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.76	0.49	0.26	0.22	0.32	0.29	0.29	0.50
0.025	1.00	0.76	0.51	0.28	0.24	0.33	0.29	0.29	0.49
0.050	1.00	0.78	0.55	0.33	0.27	0.35	0.29	0.29	0.48
0.075	1.00	0.79	0.58	0.38	0.30	0.36	0.30	0.28	0.47
0.100	1.00	0.81	0.62	0.42	0.33	0.39	0.30	0.28	0.46
0.125	1.00	0.82	0.65	0.47	0.37	0.41	0.31	0.28	0.46
0.150	1.00	0.83	0.68	0.52	0.40	0.44	0.33	0.29	0.45
0.200	1.00	0.85	0.73	0.60	0.47	0.50	0.36	0.30	0.45
0.250	1.00	0.88	0.78	0.67	0.53	0.55	0.39	0.32	0.45
0.500	1.01	0.95	0.92	0.88	0.72	0.80	0.60	0.47	0.56

Table 13. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 3 ($T=116$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_1/N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.99	0.95	0.79	0.60	0.36	0.45	0.35	0.36	0.55
0.025	0.99	0.95	0.80	0.61	0.37	0.46	0.35	0.37	0.55
0.050	0.99	0.95	0.81	0.63	0.39	0.48	0.35	0.37	0.56
0.075	0.99	0.95	0.82	0.65	0.42	0.50	0.36	0.37	0.56
0.100	0.99	0.95	0.82	0.66	0.44	0.52	0.37	0.38	0.56
0.125	0.99	0.95	0.83	0.68	0.47	0.54	0.39	0.38	0.56
0.150	0.99	0.95	0.84	0.69	0.49	0.56	0.40	0.39	0.57
0.200	0.99	0.95	0.85	0.72	0.54	0.61	0.43	0.40	0.58
0.250	0.99	0.96	0.87	0.75	0.58	0.65	0.47	0.42	0.60
0.500	0.99	0.96	0.91	0.84	0.74	0.83	0.66	0.56	0.70

Population 4

Table 14. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 4 ($T=189$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.01	0.60	0.35	0.30	0.27	0.30	0.28	0.34	0.46
0.025	1.03	0.68	0.40	0.37	0.33	0.34	0.30	0.35	0.48
0.050	1.08	0.83	0.51	0.51	0.48	0.41	0.35	0.39	0.50
0.075	1.12	0.98	0.63	0.70	0.70	0.51	0.41	0.43	0.54
0.100	1.16	1.12	0.76	0.91	1.01	0.65	0.50	0.49	0.58
0.125	1.20	1.26	0.90	1.14	1.45	0.84	0.63	0.57	0.63
0.150	1.25	1.39	1.03	1.38	2.05	1.10	0.80	0.67	0.70
0.200	1.33	1.62	1.28	1.84	3.74	1.90	1.35	0.99	0.91
0.250	1.42	1.84	1.48	2.24	5.92	3.27	2.44	1.56	1.25
0.500	1.79	2.84	2.27	4.15	12.17	14.07	101.16	44.24	16.78

Table 15. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 4 ($T=189$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.01	0.96	0.59	0.56	0.36	0.32	0.33	0.40	0.47
0.025	1.01	0.98	0.64	0.61	0.42	0.35	0.35	0.42	0.48
0.050	1.02	1.01	0.71	0.71	0.55	0.43	0.39	0.45	0.51
0.075	1.02	1.04	0.80	0.82	0.73	0.52	0.43	0.49	0.54
0.100	1.03	1.07	0.89	0.94	0.98	0.65	0.49	0.55	0.57
0.125	1.04	1.10	0.98	1.07	1.31	0.82	0.57	0.61	0.61
0.150	1.04	1.14	1.07	1.22	1.76	1.04	0.66	0.69	0.66
0.200	1.05	1.21	1.25	1.53	3.09	1.72	0.92	0.91	0.80
0.250	1.07	1.29	1.42	1.87	4.98	2.88	1.33	1.26	0.99
0.500	1.14	1.72	2.03	3.84	12.03	14.33	14.23	12.73	4.92

Table 16. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 4 ($T=189$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.00	0.58	0.34	0.30	0.26	0.31	0.29	0.36	0.51
0.025	1.01	0.62	0.36	0.33	0.28	0.32	0.29	0.36	0.50
0.050	1.01	0.67	0.41	0.38	0.33	0.33	0.30	0.35	0.48
0.075	1.02	0.72	0.45	0.44	0.37	0.35	0.30	0.35	0.47
0.100	1.02	0.76	0.49	0.50	0.43	0.37	0.31	0.34	0.45
0.125	1.03	0.79	0.53	0.56	0.48	0.39	0.33	0.35	0.44
0.150	1.03	0.82	0.57	0.61	0.53	0.42	0.34	0.35	0.43
0.200	1.04	0.87	0.63	0.70	0.62	0.48	0.38	0.37	0.43
0.250	1.04	0.90	0.69	0.77	0.71	0.54	0.43	0.40	0.43
0.500	1.06	1.01	0.85	0.98	1.00	0.79	0.67	0.60	0.55

Table 17. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 4 ($T=189$, Area= 9100 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	1.01	0.96	0.59	0.56	0.35	0.33	0.34	0.42	0.51
0.025	1.02	0.97	0.60	0.58	0.37	0.33	0.34	0.42	0.50
0.050	1.02	0.97	0.63	0.61	0.41	0.35	0.35	0.42	0.50
0.075	1.02	0.97	0.66	0.64	0.44	0.37	0.35	0.42	0.49
0.100	1.02	0.98	0.68	0.67	0.49	0.39	0.36	0.42	0.49
0.125	1.02	0.98	0.71	0.70	0.53	0.41	0.38	0.43	0.49
0.150	1.02	0.99	0.73	0.72	0.58	0.44	0.39	0.44	0.49
0.200	1.02	1.00	0.77	0.77	0.66	0.49	0.42	0.45	0.50
0.250	1.02	1.00	0.80	0.82	0.74	0.55	0.45	0.48	0.51
0.500	1.02	1.03	0.89	0.97	1.02	0.80	0.65	0.65	0.65

Population 5

Table 18. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 5 ($T=625$, Area= 6000 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.70	0.38	0.46	0.46	0.35	0.42	0.32	0.59	0.49
0.025	0.86	1.02	0.79	0.62	0.42	0.45	0.33	0.60	0.50
0.050	1.15	5.37	2.02	1.05	0.60	0.53	0.36	0.62	0.52
0.075	1.44	22.76	5.30	1.84	0.89	0.62	0.40	0.65	0.54
0.100	1.69	53.22	13.91	3.32	1.35	0.76	0.44	0.68	0.57
0.125	1.89	72.56	34.17	6.14	2.11	0.94	0.50	0.72	0.59
0.150	2.05	81.74	70.75	11.59	3.38	1.19	0.57	0.78	0.63
0.200	2.30	92.68	146.17	42.38	9.37	2.02	0.79	0.92	0.72
0.250	2.51	100.80	180.94	137.40	28.30	3.70	1.15	1.15	0.84
0.500	3.57	126.78	279.84	759.96	1562.27	249.25	21.00	7.93	3.32

Table 19. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 5 ($T=625$, Area= 6000 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.80	0.43	0.48	0.49	0.43	0.52	0.34	0.72	0.57
0.025	0.92	0.90	0.79	0.64	0.51	0.56	0.36	0.74	0.58
0.050	1.14	3.05	1.89	1.03	0.69	0.65	0.38	0.77	0.61
0.075	1.34	8.65	4.60	1.68	0.94	0.77	0.42	0.81	0.64
0.100	1.52	17.11	11.23	2.81	1.31	0.92	0.46	0.86	0.68
0.125	1.66	23.96	26.20	4.78	1.87	1.11	0.51	0.91	0.72
0.150	1.79	28.25	53.62	8.26	2.70	1.37	0.58	0.97	0.77
0.200	1.99	34.05	116.82	25.23	5.97	2.16	0.75	1.13	0.89
0.250	2.17	39.23	144.01	70.68	14.13	3.62	1.03	1.35	1.06
0.500	3.09	67.32	182.20	254.70	576.57	129.60	11.20	5.72	4.26

Table 20. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 5 ($T=625$, Area= 6000 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.65	0.26	0.38	0.41	0.33	0.42	0.33	0.61	0.51
0.025	0.70	0.36	0.46	0.45	0.34	0.42	0.32	0.59	0.50
0.050	0.78	0.53	0.59	0.52	0.37	0.42	0.32	0.57	0.49
0.075	0.83	0.68	0.72	0.59	0.40	0.42	0.31	0.55	0.48
0.100	0.88	0.79	0.83	0.65	0.44	0.43	0.31	0.54	0.47
0.125	0.91	0.87	0.92	0.71	0.48	0.44	0.31	0.52	0.46
0.150	0.94	0.94	1.00	0.77	0.52	0.46	0.31	0.51	0.46
0.200	0.98	1.05	1.12	0.86	0.60	0.50	0.31	0.50	0.44
0.250	1.01	1.12	1.22	0.93	0.67	0.55	0.33	0.49	0.44
0.500	1.08	1.32	1.48	1.18	0.96	0.84	0.46	0.61	0.46

Table 21. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 5 ($T=625$, Area= 6000 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_i / N	Sample Unit Length (m)								
	1	2	3	4	5	7.5	10	12.5	15
0.010	0.75	0.32	0.41	0.45	0.42	0.52	0.35	0.74	0.59
0.025	0.79	0.42	0.48	0.49	0.43	0.53	0.35	0.73	0.58
0.050	0.84	0.57	0.61	0.56	0.47	0.53	0.34	0.72	0.58
0.075	0.88	0.70	0.74	0.63	0.50	0.54	0.34	0.72	0.57
0.100	0.91	0.79	0.85	0.70	0.53	0.56	0.34	0.71	0.57
0.125	0.93	0.87	0.95	0.76	0.57	0.57	0.34	0.70	0.56
0.150	0.95	0.93	1.03	0.81	0.60	0.59	0.34	0.70	0.56
0.200	0.98	1.02	1.15	0.90	0.66	0.63	0.35	0.69	0.55
0.250	1.00	1.08	1.24	0.97	0.72	0.68	0.36	0.70	0.55
0.500	1.05	1.24	1.48	1.17	0.92	0.93	0.48	0.81	0.59

Population 6

Table 22. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 6 ($T=1997$, Area= 220,400 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_1/N	Sample Unit Length (m)											
	1	2	3	4	5	6	7	8	9	10	12.5	15
0.010	1.04	1.02	0.86	0.65	0.60	0.54	0.50	0.53	0.50	0.51	0.42	0.39
0.025	1.07	1.15	1.15	1.09	1.00	0.93	0.83	0.85	0.74	0.73	0.57	0.50
0.050	1.10	1.37	1.66	1.83	1.79	1.70	1.53	1.63	1.33	1.28	0.90	0.76
0.075	1.13	1.56	2.15	2.57	2.66	2.54	2.38	2.61	2.10	2.03	1.34	1.11
0.100	1.16	1.72	2.60	3.32	3.64	3.48	3.43	3.76	3.00	2.98	1.87	1.55
0.125	1.18	1.85	3.00	4.03	4.71	4.52	4.75	5.10	4.03	4.11	2.47	2.07
0.150	1.20	1.96	3.33	4.66	5.80	5.60	6.41	6.65	5.22	5.45	3.15	2.64
0.200	1.23	2.13	3.86	5.64	7.85	7.74	10.73	10.37	8.16	9.03	4.77	3.84
0.250	1.26	2.27	4.27	6.35	9.57	9.70	15.94	14.66	11.90	14.18	6.97	5.11
0.500	1.37	2.78	5.66	8.52	14.68	16.54	35.49	35.29	37.56	55.92	37.36	13.08

Table 23. The relative efficiency of the ACS HT estimator to the SRS estimator of the total for Population 6 ($T=1997$, Area= 220,400 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_1/N	Sample Unit Length (m)											
	1	2	3	4	5	6	7	8	9	10	12.5	15
0.010	1.02	1.04	0.98	0.81	0.69	0.65	0.57	0.54	0.54	0.54	0.51	0.47
0.025	1.03	1.08	1.10	1.02	0.90	0.94	0.84	0.83	0.77	0.74	0.65	0.58
0.050	1.04	1.15	1.33	1.44	1.32	1.50	1.36	1.51	1.28	1.19	0.96	0.82
0.075	1.05	1.21	1.57	1.95	1.85	2.11	1.94	2.36	1.93	1.78	1.36	1.11
0.100	1.06	1.27	1.82	2.51	2.46	2.74	2.59	3.35	2.66	2.49	1.85	1.45
0.125	1.06	1.33	2.05	3.07	3.15	3.42	3.35	4.50	3.48	3.30	2.40	1.83
0.150	1.07	1.38	2.26	3.59	3.88	4.16	4.25	5.83	4.39	4.22	3.00	2.23
0.200	1.09	1.46	2.62	4.45	5.35	5.73	6.59	9.06	6.62	6.55	4.41	3.10
0.250	1.10	1.54	2.88	5.09	6.61	7.29	9.58	12.88	9.41	9.78	6.21	4.09
0.500	1.15	1.85	3.66	6.89	9.36	12.48	24.79	28.43	26.19	34.61	27.29	12.05

Table 24. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 6 ($T=1997$, Area= 220,400 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_1 \{i: y_i > 0\}$.

n_1/N	Sample Unit Length (m)											
	1	2	3	4	5	6	7	8	9	10	12.5	15
0.010	1.03	0.96	0.77	0.52	0.49	0.44	0.42	0.45	0.43	0.45	0.39	0.37
0.025	1.04	1.01	0.87	0.66	0.61	0.57	0.52	0.55	0.51	0.52	0.45	0.41
0.050	1.05	1.07	0.99	0.82	0.77	0.73	0.66	0.69	0.64	0.63	0.55	0.48
0.075	1.05	1.11	1.07	0.93	0.87	0.85	0.77	0.81	0.74	0.73	0.64	0.55
0.100	1.06	1.14	1.12	1.00	0.96	0.95	0.86	0.91	0.83	0.81	0.72	0.62
0.125	1.06	1.16	1.17	1.06	1.02	1.02	0.93	0.98	0.90	0.88	0.79	0.68
0.150	1.06	1.18	1.20	1.11	1.07	1.08	0.99	1.05	0.95	0.94	0.84	0.74
0.200	1.06	1.20	1.25	1.18	1.15	1.18	1.08	1.15	1.05	1.03	0.93	0.84
0.250	1.07	1.22	1.28	1.23	1.21	1.25	1.16	1.23	1.12	1.11	1.01	0.92
0.500	1.08	1.27	1.37	1.37	1.36	1.44	1.37	1.46	1.32	1.34	1.22	1.17

Table 25. The relative efficiency of the ACS HH estimator to the SRS estimator of the total for Population 6 ($T=1997$, Area= 220,400 m²) with sample unit lengths ranging from 1 m to 15 m and initial sampling fractions ranging from 0.01 to 0.50. The condition to include additional units in a sample is $C_2 \{i: y_i > 1\}$.

n_1/N	Sample Unit Length (m)											
	1	2	3	4	5	6	7	8	9	10	12.5	15
0.010	1.02	1.02	0.93	0.74	0.63	0.56	0.50	0.47	0.48	0.49	0.48	0.45
0.025	1.02	1.03	0.95	0.79	0.69	0.66	0.60	0.56	0.56	0.56	0.53	0.49
0.050	1.02	1.04	0.99	0.86	0.78	0.79	0.72	0.70	0.68	0.66	0.62	0.56
0.075	1.02	1.05	1.03	0.92	0.85	0.88	0.81	0.82	0.78	0.76	0.70	0.63
0.100	1.02	1.06	1.05	0.97	0.91	0.94	0.88	0.91	0.86	0.83	0.77	0.69
0.125	1.02	1.06	1.07	1.00	0.96	1.00	0.93	0.99	0.92	0.90	0.83	0.75
0.150	1.02	1.07	1.09	1.03	1.00	1.04	0.98	1.05	0.98	0.95	0.89	0.80
0.200	1.02	1.08	1.11	1.08	1.06	1.11	1.06	1.14	1.07	1.04	0.97	0.89
0.250	1.02	1.08	1.13	1.11	1.11	1.16	1.11	1.22	1.13	1.10	1.04	0.96
0.500	1.02	1.10	1.18	1.20	1.22	1.30	1.27	1.41	1.31	1.28	1.24	1.18