VARIANCE ESTIMATION FOR ADAPTIVE CLUSTER SAMPLING WITH A SINGLE PRIMARY UNIT AND THE PARTIALLY SYSTEMATIC ADAPTIVE CLUSTER SAMPLING

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A Dissertation Submitted in Partial

Fulfillment of the Requirements for the Degree of

Doctor of Philosophy (Statistics)

School of Applied Statistics

National Institute of Development Administration

2009

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ABSTRACT

Title of Dissertation	Variance Estimation for Adaptive Cluster Sampling with a Single Primary Sampling Unit and the Partially Systematic Adaptive Cluster Sampling
Author	Miss Urairat Netharn
Degree	Doctor of Philosophy (Statistics)
Year	2009

Two topics are investigated in this dissertation. The first concerns variance estimation when a single primary sampling unit is selected. Two new bias variance estimators, based on splitting the initial sample into sub-samples and regarding the initial sample as a stratified sample, are proposed. The results of this study indicated that both new variance estimators are underestimated. The first variance estimator is not preferable when the number of sub-samples is two because its relative bias is too large to be useful. Increasing the number of sub-samples made its relative bias decrease. When the sub-sample size and stratum size equal two, the second variance estimator is more efficient than the first in terms of minimum relative bias and mean squared error. However, both new variance estimators are less efficient than non-adaptive variance estimators in systematic sampling in terms of relative bias in some case. This may result from ignoring the correlation terms between sub-samples or between units in stratum. In addition, the percentage of intervals containing the true population mean for both new variance estimators is less than ninety-five percent.

For the second topic, the design of partially systematic adaptive cluster sampling, in which the initial sample selected by sampling without replacement of units, without replacement of networks, and without replacement of clusters, is studied. The results of this study indicated that all three sampling procedures can provide unbiased estimators of the population mean and its variance. An unbiased estimator of the population mean based on a selection without replacement of clusters is the most efficient in terms of minimum variance, while an unbiased estimator of the population mean obtained by sampling without replacement of units is the least efficient. The efficiency comparison between all three estimators proposed for partially systematic adaptive cluster sampling and a modified Raj type estimator proposed by Raj (1956) indicated that the proposed estimators are more efficient than the modified Raj type estimator. However, the percentage of intervals containing the true population mean for the proposed estimators is less than ninety-five percent. This may have been caused by the distribution of proposed estimators not being asymptotic to normal distribution.

ACKNOWLEDGEMENTS

I would like to express my gratitude to all those who gave me the possibility to complete this research. I want to thank and offer my appreciation to my advisor, Assistant Professor Dr. Arthur L. Dryver, for his wisdom, knowledge, guidance and encouragement—he helped throughout the research and the writing of this report.

I have furthermore to thank Professor Dr. Prachoom Suwattee, Associated Professor Dr. Surin Niyamangkoon and Associated Professor Dr. Samruam Chongcharoen for their constructive comments and helpful advice.

I would also like to offer special thanks to Dr. Bruce Leeds, whose help in correcting my English has made the manuscript more readable, and to Dr. Nipaporn and Mr. Paveen Chutiman, who advised me on issues of programming. Appreciation is also extended to all lecturers and staff in the School of Applied Statistics at the National Institute of Development Administration.

I would like to thank my friends in the Department of Statistics for their cheerfulness and for making my study at the institute a rewarding experience of learning and friendship.

Especially, I would like to give my special thanks to my parents, my sister and my brother's family, whose support and patient love enabled me to complete this work.

Urairat Netharn

March 2009

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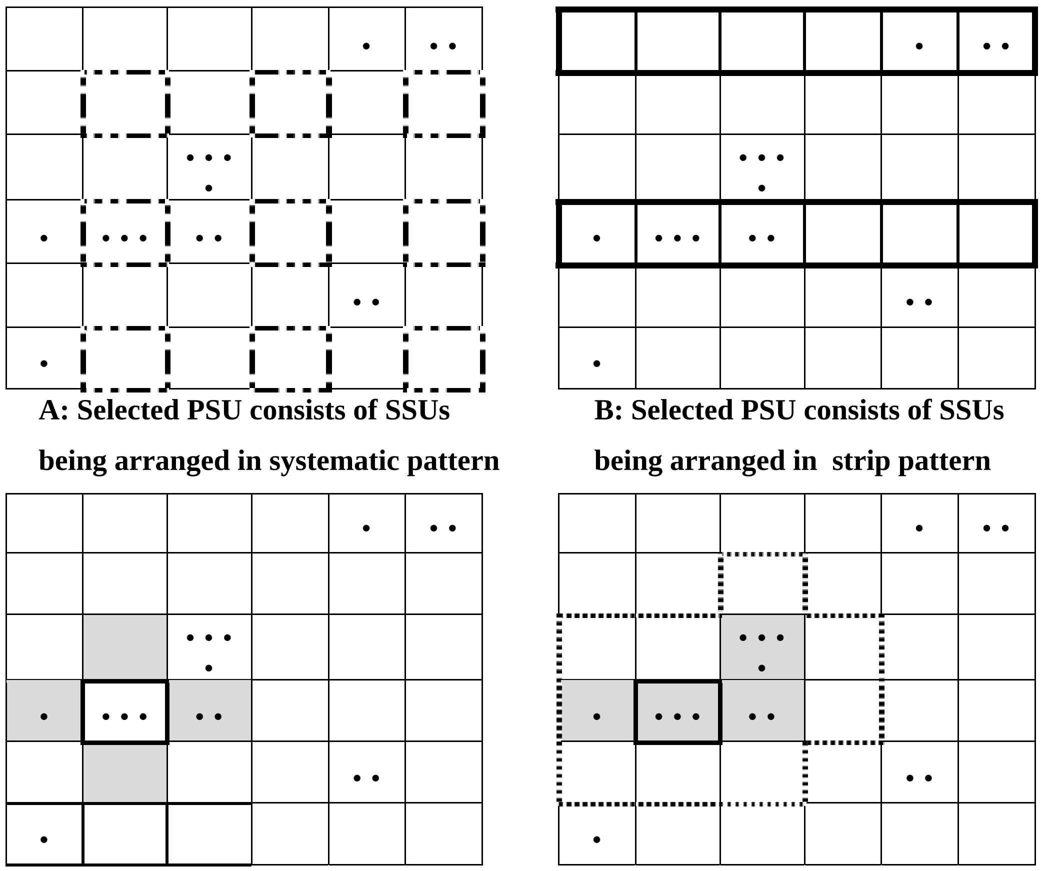
CHAPTER 1

INTRODUCTION

1.1 Background

Adaptive cluster sampling (ACS) proposed by Thompson (1990: 1050-1059) is a sampling design that is suitable for a rare and clustered population or rare characteristic of interest such as heroin use or HIV infection and species density or abundance etc. However, in many surveys, especially for biological populations such as birds, fish, and tree species etc. the selection site is often done systematically. Thompson (1991: 1103-1115) developed an adaptive sampling design by using systematic sampling to select the initial sample. The initial sample is defined as a collection of primary sampling units sampled. A primary sampling unit (PSU) is composed of units that are called secondary sampling units (SSUs). Secondary sampling units within any primary sampling unit may arrange in horizontal strip, vertical strip or systematic pattern. Figure 1A shows the area of 36 meter squares in survey of fresh water mussel diversity. This study area is divided into 36 plots with each of size 1 meter-squares. A dot within each plot represents a number of mussels counted and a case of a single primary sampling unit chosen is considered. In figure 1.1A, the selected primary sampling unit is composed of 9 secondary sampling units being arranged in systematic pattern while the selected primary sampling unit consisting of 12 secondary sampling units being arranged in strip pattern is shown in figure 1.1B. Whenever a secondary sampling unit satisfies the precondition which is defined by researcher, the unit in its neighborhood will be added to the sample. A neighborhood of a unit is defined such that if unit u_{ij} is in the neighborhood of unit $u_{i'j'}$

then unit $u_{i'j'}$ is also in the neighborhood of unit u_{ij} . For this study a neighborhood of a unit will be defined as a set of surrounding units in North, South, East and West. In figure 1.1, the precondition is defined by detection of equal or greater than 1 mussel in a plot. If any other secondary sampling units that satisfy the condition are additively added to the sample, then their neighborhood is also added to the sample; an example of a neighborhood of a unit is shown in figure 1.1C. This procedure is continued until no more secondary sampling units are found that satisfy the condition. A secondary sampling unit not satisfying the condition, but one or more units in its neighborhood satisfying the condition, is called an edge unit. A set of secondary sampling units that satisfies the condition is called a network (see figure 1.1D). The combination of a network and its associated edge units makes up a cluster, as shown in figure 1.1D, where edge units and all units within the dash lines represent a cluster.



A: Selected PSU consists of SSUs being arranged in systematic pattern

B: Selected PSU consists of SSUs being arranged in strip pattern

C: Network

D: Cluster

Figure 1.1 An Example of the Selected Initial Sample from Adaptive Cluster Sampling with a Single Primary Sampling Unit.

The application of the design is described in a sample of rare tree species in Nepal (Acharya et al., 2000: 65-73). According to their study, adaptive design is more efficient for density estimation than conventional systematic sampling in the case of clustered species. Another application is performed in a survey of North American freshwater mussels (Bivalvia: Unionidae) which tends to be extinction species in the Cacapon River, West Virginia (Smith et al., 2003: 7-15). Their results showed that adaptive cluster sampling for fixed sample size does not decrease a sampling error when the sample size is equal to the final sample. Adaptive cluster sampling makes the number of individual samples and the probability of detecting a rare species increase. However, this adaptive design is done when the initial sample has a size of at least two primary sampling units because an estimator for variance is not available in the case where a single primary sampling unit is selected by simple random sampling and where secondary sampling units are selected by “systematic” method. This specific case for adaptive cluster sampling will be investigated as in practice this is not uncommon for conventional sampling. Presently, for adaptive cluster sampling, an estimator of the variance of the estimate of the population mean or total does not exist when only one primary sampling unit is selected (Thompson, 1992: 292-296).

For conventional sampling, several methods of estimation of variance are reviewed and theoretical compared in Cochran (1977: 212-226) and Wolter (1984: 781-790, 2003: 248-305). According to the study of Wolter (1984: 781-790, 2003: 248-305) one variance estimator might be preferable in a real population where the characteristic type is linear trend or stratification effect. This variance estimator obtained by regarding the sample as a stratified sample. Though it is biased its bias often smaller than the other estimators. Another interesting variance estimator is the estimator obtained from spiting the sample into sub-samples. Though its bias is very large when the number of sub-samples is two (Koop, 1971: 1084-1087; Wolter, 1984: 781-790), its form is simple for application and its bias be reduced when the number of sub-samples is increased (Wolter, 1984: 781-790).

In addition, a procedure called partially systematic sampling was proposed by Zinger (1980: 206-211), which selects a systematic sample of size m and selects a supplementary sample of n units by simple random sampling from the remaining $N-m$ units in the population; this was proposed to estimate the mean of a finite population and to estimate the variance of this estimate when the population size N is a multiple of sample size and the sampling interval is k. This method provides unbiased estimators. The purpose of this sampling procedure is to propose a design that is cost effective, as well as systematic, but that obtains an unbiased estimator for the variance when only a single primary sampling unit (a systematic sample) is selected with all of the secondary sampling units. This is done by taking a simple random sample from the remaining units. The new partially systematic sampling procedure proposed by Leu and Tsui (1996: 617-630) does not required the condition of the population size N that has to be a multiple of sample size or sampling interval and this procedure can provide an unbiased estimator of the population mean and variance.

This study will cover variance estimators obtained based on the above two methods of variance estimation, where adaptive cluster sampling is done and for which only one primary sampling unit has been selected. In addition, estimators concerned with the modification of the concept of partially systematic sampling to be partially systematic adaptive cluster sampling will be proposed. The properties of proposed estimators in each part will be investigated by using simulation.

1.2 Objectives of the Study

The objectives of the study are as follows:

1.2.1 To find two new biased estimators for the variance of an estimator of population mean proposed by Thompson where a single primary sampling unit is selected.

1.2.2 To propose an estimator of the population mean and its variance for design of partially systematic adaptive cluster sampling.

1.3 Scope of the Study

The scope of the study is as follows:

1.3.1 Derivation of two new biased estimators for the variance of an unbiased estimator for the population mean proposed by Thompson for ACS when a single primary sampling unit (single PSU) is selected. Consequently of the complicated formulae of the two new variance estimators makes theoretical comparison very difficult to impossible. Thus their properties will be investigated by using simulation study. These two new estimators of variance will be compared to each other empirically in terms of:

1) Bias of the estimator of variance

2) MSE of the biased estimator of the variance

3) Percent of intervals containing the true mean for a 95% hypothesized confidence interval.

1.3.2 Derivation of an estimator of the population mean and its variance for the design of partially systematic adaptive cluster sampling.

1) The newly-derived associated estimator for the population mean for the design of partially systematic adaptive cluster sampling will be compared in terms of efficiency to the conventional estimator for the population mean for non-adaptive sampling with a comparable final sample size.

2) The newly-derived estimator for the variance of the estimator for the population mean and the biased variance estimators derived for part A in terms of simulated percent capture true mean using an assumed 95% confidence interval.

3) For the theoretical properties comparison in the previous, consequently of the complicated formulae of the proposed estimators makes the mathematics comparison very difficult to impossible. Thus the efficiency comparison of proposed estimators of population mean and variance estimator derived for this design will be investigated by using simulation study.

1.4 Methodology

1.4.1 For Adaptive Cluster Sampling with a Single Primary Sampling Unit

When a single primary sampling unit (a single PSU) is selected, an unbiased estimator of the population mean can be obtained. A modified Hansen-Hurwitz's type estimator which is an unbiased estimator of the population mean and proposed by Thompson (1991: 1103-1115) is of interest in this study. The methodologies for estimation of the variance when a single primary sampling unit is selected focuses on two methods of variance estimation for systematic sampling. These two methods are applied to the initial sample or the selected primary sampling unit for adaptive cluster sampling. In the first method of estimation, the concept of splitting the sample into sub-samples is used for finding the variance estimator. The value of the number of sub-samples is specified with a value of at least two sub-samples. Each sub-sample consists of at least two secondary sampling units (SSUs). Next, for the remaining method of estimation, the initial sample is treated as if it were a stratified random sample, with the number of strata being more than or equal to two. The number of units within each stratum is specified with only two secondary sampling units. Properties and efficiency comparison of both new variance estimators are investigated using simulation. The simulation study has been carried out with a small population and real data of blue-winged teal which is given by Smith at el. (1995: 777-778).

1.4.2 For the Partially Systematic Adaptive Cluster Sampling (Partially SACS)

The methodologies for finding the proposed new estimators for partially SACS are modified based on the concept of partially systematic sampling. In partially SACS, the single primary sampling unit and the number of supplementary samples are selected at random. These units are considered in terms of the initial sample. The supplementary samples are composed of a number of secondary sampling units that are not the elements of the selected primary sampling unit. The supplementary sample size of interest is the sample containing at least one secondary sampling unit but is not equal to or greater than the number of secondary sampling units in each primary sampling unit.

The scheme of the selection of units for the initial sample is considered in case of varying probability of selection. Three sampling procedures for the selection are investigated: sampling without replacement of units, sampling without replacement of networks, and sampling without replacement of clusters. Details of each sampling procedure are shown as follows:

1.4.2.1 For Sampling Without Replacement of Units

The beginning step of sampling without replacement of units is the selection of a single primary sampling unit from total N primary sampling units in the population. This selected primary sampling unit is called the first initial unit. When any secondary sampling units meet the precondition, all corresponding networks and their associated edge units in which any secondary sampling units of the first belongs are created and added to the sample. For the second selection, a secondary sampling unit is selected from the population exclusive of all of secondary sampling units within the previous initial unit. The selected secondary sampling unit is called the second initial unit. Its associated network and edge unit are created and are added to the sample. Only the second initial unit is removed before the next selection is made. This procedure is repeated until all of the initial units have been selected. According to this procedure of the selection, none of the selected secondary sampling units in the previous selection can be selected repeatedly.

1.4.2.2 For Sampling Without Replacement of Networks

The procedure of sampling without replacement of networks also began with the selection of a single primary sampling unit, or the first initial unit. Whenever any secondary sampling units in the first initial unit satisfy the precondition, all corresponding networks and their associated edge units are created and are added to the sample. For the second selection, a secondary sampling unit is selected from the population exclusive of all networks obtained from the previous initial unit. Its associated network and edge units are created and added to the sample. This network is removed from the population before the next selection. This procedure is repeated until all of the initial units have been selected. Under this sampling procedure of the selection, any networks obtained from the previous selection cannot be selected repeatedly.

1.4.2.3 For Sampling Without Replacement of Clusters

For sampling without replacement of clusters, the first initial unit is selected with the same procedure as the sampling without replacement of networks. But the additional unit is different. For the sampling without replacement of clusters, whenever any units satisfy the condition, its associated cluster, which is composed of a network and its associated edge units, will be added to the sample. This procedure is repeated until all of the initial units have been selected. These clusters will be deleted from the population before the next selection will be performed. According to this procedure of the selection, any selected network and its associated edge units cannot be repeatedly selected.

The estimator of the population mean and its variance corresponding to each sampling scheme is carried out based on a modified Raj's estimator. Only information of networks is used in the calculation of proposed estimator of the population mean and its variance when a selection is without replacement of units and without replacement of networks while information of clusters is used in the calculation of proposed estimators of the population mean and its variance when a selection is without replacement of clusters. Properties and efficiency comparison of these proposed estimators are investigated using simulation. The simulation study has been carried out with a small population and real data of blue-winged teal which is given by Smith at el. (1995: 777-778).

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The adaptive cluster sampling design (ACS), with primary and secondary sampling units, was proposed by Thompson (1991: 1103-1115) as an alternative sampling strategy for estimating totals and means of rare and cluster populations. This type of population is often found in environmental or biological populations and, in general, a number of primary sampling units (more than one primary sampling unit) are taken at random.

One design-unbiased estimator of the population mean and its variance proposed by Thompson (1991: 1103-1115) is a modified Hansen-Hurwitz type estimator. This estimator required an initial sample with a size of at least two primary sampling units. In addition, Thompson (1992: 295) states the following concerning when only a single primary sampling unit is selected: "For an initial systematic sample with only one starting point (i.e., only one primary sampling unit is selected), some of the joint inclusion probabilities are zero, underscoring the fact that an unbiased estimator of variance is not available for such a design." But the above estimator of the population mean is unbiased.

In this chapter, adaptive cluster sampling, variance estimation in conventional systematic sampling when a systematic sample is selected, partially systematic sampling, and adaptive cluster sampling without replacement of networks or clusters, will be discussed.

2.2 Adaptive Cluster Sampling

For the adaptive cluster sampling design with primary and secondary sampling units, the selected primary sampling units will be called the initial sample.
For estimating the population mean, a modified Hansen-Hurwitz type of estimator proposed by Thompson (1991: 1103-1115) is discussed as follow:

2.2.1 The Modified Hansen-Hurwitz Type of Estimator

A finite rare and clustered population consists of N primary sampling units, with each primary sampling unit (PSU) containing M secondary sampling units. Let u_{ij} denote the j^{th} secondary sampling unit of the i^{th} primary sampling unit with the associated y-value of y_{ij}. The draw-by-draw probabilities of the unit are the probability of selecting a single primary sampling unit associated with any network containing unit u_{ij} or any network not containing unit u_{ij} but unit u_{ij} is in the neighborhood of its elements.

A modified Hansen-Hurwitz type estimator for the population mean proposed by Thompson (1991: 1108-1109) can be written as

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\hat{\mu}_i\ , \qquad (2.1)$$

where n is the number of primary sampling units selected, $\hat{\mu}_i = \left(\sum_{k=1}^{\kappa_i} y_k / x_k\right)\Big/ M$, y_k is the total y-values of the k^{th} network, x_k is the number of primary sampling units corresponding to the k^{th} network, and κ_i is the number of networks associated with the i^{th} primary sampling unit.

The variance of $\hat{\mu}$ is

$$V(\hat{\mu}) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{N}(\hat{\mu}_i - \mu)^2}{N-1} \tag{2.2}$$

An unbiased estimator of $V(\hat{\mu})$ is

$$\hat{v}(\hat{\mu}) = \frac{N-n}{Nn}\frac{\sum_{i=1}^{n}(\hat{\mu}_i - \hat{\mu})^2}{n-1} \tag{2.3}$$

Note that the initial sample consists of more than one primary sampling unit, or $n \geq 2$.

2.2.2 Asymptotics in Adaptive Cluster Sampling

The asymptotic property of the modified Hansen-Hurwitz (HH) type estimator proposed by Thompson in 1990 was studied by Felix-Medina (2003: 61-82). According to his study the condition of the tendency to infinity of the number of units in the initial sample and the units and networks and the condition of the boundary of the network size are established in order to investigate the asymptotic normality and consistency of the HH type estimator. His result showed that both an HH type of estimator and its variance are a design-consistent estimator.

2.3 Variance Estimation in Conventional Systematic Sampling When a Single Primary Sampling Unit is Taken

2.3.1 Variance Estimation When a Single Primary Sampling Unit (a Systematic Sample) is Chosen

In conventional systematic sampling, let NM denote the population size, N denote the number of primary sampling units (PSU), M denote the number of secondary units within each primary sampling unit, and n denote the number of primary sampling units in the sample. When a single primary sampling unit is selected, that is $n=1$, the sampling interval is $q = NM/nM = N$, there are possible N primary sampling units to selected from the population. The single primary sampling unit is selected at random with probability $1/N$. Suppose the i^{th} primary sampling unit is chosen; let Y_{ij} be the y-value of the j^{th} units in the i^{th} primary sampling unit, where $i = 1, 2, ..., N$ and $j = 1, 2, ..., M$. When a single primary sampling unit is chosen, an unbiased estimator of the population mean is (Wolter, 1984: 781-782).

$$\bar{y}_{sys(i)} = \frac{1}{M}\sum_{j=1}^{M} y_{ij} \,, \qquad (2.4)$$

where i represent the primary sampling unit selected.

The variance of $\bar{y}_{sys}$ is

$$V\left(\bar{y}_{sys}\right) = \left(\sigma^2/M\right)\left[1 + \left(M-1\right)\rho\right] \,, \qquad (2.5)$$

where $\sigma^2 = \sum_{i=1}^{N}\sum_{j=1}^{M}\left(Y_{ij} - \bar{Y}_{\bullet\bullet}\right)^2 \Big/ MN$ is the population variance, $\bar{Y}_{\bullet\bullet}$ is the population mean, and $\rho = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{j'\neq j}^{M}\left(Y_{ij} - \bar{Y}_{\bullet\bullet}\right)\left(Y_{ij'} - \bar{Y}_{\bullet\bullet}\right) \Big/ MN\left(M-1\right)\sigma^2$ is the intraclass correlation between pairs of units in the same PSU sample.

Eight biased variance estimators were shown as alternative estimators of the variance, $V\left(\bar{y}\right)$, in Cochran (1977: 212-226) and Wolter (1984: 781-790 , 2003: 248-305). The theoretical properties and Monte Carlo method were performed to investigate the properties of these estimators. Two of eight biased estimators are performed based on the method of splitting the sample into a sub-sample and treating the sample as a stratified random sample. In the first method, the selected primary sampling unit with size $M = mp$ is divided at random into p sub-samples with size $m = M/p$ units, but only the value of $p = 2$ is performed in the study of Wolter (1984: 781-790). For the second method, the selected primary sampling unit is treated as it is a stratified sample with $p = M/2$ strata. Each stratum is composed of M_t units and $M = \sum_{t=1}^{p} M_t = 2p$. The sample of size $m = 2$ units is selected at random from each stratum. Both methods for estimation provide two biased variance estimators and their details are as follows:

Method I: $M = mp, p \geq 2$ and $m \geq 2$.

$$\hat{v}_1\left(\bar{y}_{sys}\right) = \left(\frac{N-1}{N}\right)\frac{1}{p}\frac{\sum_{t=1}^{p}\left(\bar{y}_t - \bar{y}\right)^2}{p-1} , \tag{2.6}$$

where $\bar{y}_t$ represents the sample mean of the t^{th} sub-sample of size $m = M/p$ and $\bar{y}$ represents the systematic sample mean.

Method II: $M = 2p, p \geq 2$ and $m = 2$.

$$\hat{v}_2\left(\bar{y}_{sys}\right) = \left(\frac{N-1}{N}\right)\frac{1}{M}\frac{\sum_{j=1}^{M/2}\left(y^*_{2j} - y^*_{2j-1}\right)^2}{M} , \tag{2.7}$$

where y^*_{2j} denotes the y-value of the $2j^{th}$ unit.

Three properties of a variance estimator in terms of bias, mean squared error, and the proportion of the confidence interval that contained the true population mean were performed with the Monte Carlo simulation (Wolter, 1984: 781-790, 2003: 248-305). Several types of characteristics of the populations (linear trend, stratification effects) were considered, and a 200 finite population of size NM = 1,000 was studied. According to the Monte Carlo simulation study, the variance estimator obtained by dividing the sample into sub-samples is not recommended to be used in practice because of the large value of bias when the number of sub-samples equals two, but its bias may be reduced when the number of sub-samples is increased. The bias of the variance estimator obtained by regarding the sample as a stratified sample is often smaller than the other variance estimators for several types of characteristics of the population, such as linear trend, stratification effects, etc.

2.4 Partially Systematic Sampling

In conventional systematic sampling, a method of composition is often used to handle the problem of variance estimation is a method of composition. In this method, the sample size M is split into two parts, m_0 and $M - m_0$, where $m_0 \geq 1$. The sample of size $M - m_0$, which is a fixed-size, is selected by systematic sampling, while another sample is drawn from remaining units in population by adopting any sampling design. The two components samples together constitute the ultimate sample of size M (Hedayat and Sinha, 1991: 239-241).

Partially systematic sampling, a new sampling procedure, was proposed by Zinger in 1980 in order to handle the problem of variance estimation. In this sampling procedure, the population with a size of NM units, which is equal to the cross-product of sample size and the sampling interval, is considered, where a sampling interval is $q = NM/nM = N$ when a single primary sampling unit is selected, $n = 1$. In this sampling procedure, a single primary sampling unit with size M units is selected at random and then a supplementary sample is drawn from $NM - M$ units in the population. The supplementary sample can be chosen either by systematic sampling or simple random sampling. This sampling procedure provides an unbiased estimator of the population mean and its variance. The unbiased estimator is obtained by the weight of the sample mean of the selected PSU and the sample mean of the supplementary sample. In addition, the optimal value of weight is finding out. According to his study the recommend weighted is the equal weighted.

The new sampling procedure of the selection for partially systematic sampling was proposed by Leu and Tsui (1996: 617-630) and is called the "new partially systematic sampling." According to their study, this sampling procedure can be applied with a population whose size is not equal to the multiple of the sample size and the sampling interval. This new sampling procedure can provide both unbiased estimators for the population mean and its variance. In addition, the "new partially systematic sampling" is more efficient than other sampling procedures: simple random sampling, systematic sampling with multiple random starts for population with "random", "linear trends," and "periodic variation".

The sampling procedure of the unit selection can be performed with the other sampling procedure of the selection. For conventional sampling, the sampling procedure often used when unit sampled with varying probabilities was proposed by Raj (1956: 269-284). This sampling procedure is called "sampling without replacement of units." The details have been summarized below.

2.5 Sampling Without Replacement of Units

For conventional sampling, in a sampling of n units selected without replacement from the finite population, the probability of any unit chosen in any draw depends on the units already drawn in the sample, not on the order in which they are drawn. Let notation i represent the sample unit obtained from the i^{th} selection, p_1 be the first draw probability of units for $i=1$, and the conditional probability of unit chosen on the i^{th} draw given $i-1$ units have been drawn be $p_i \Big/ \left(1-\sum_{i=1}^{i-1} p_i\right)$, where $i = 2,3,\dots,n$.

Under this scheme of selection, an unbiased estimator of the population total (Raj, 1956: 271-272) is

$$\hat{t} = \frac{1}{n}\sum_{i=1}^{n} t_i \quad , \tag{2.8}$$

where $t_1 = y_1 / p_1$ for $i = 1$ and $t_n = \sum_{i=1}^{n-1} y_i + \left\{ y_n \left(1 - p_1 - ... - p_{n-1}\right) / p_n \right\}$ for $i = 2, 3, ..., n$.

The variance of $\hat{t}$ is

$$V\left(\hat{t}\right) = \frac{1}{n^2}\sum_{i=1}^{n} V\left(t_i\right) \tag{2.9}$$

An unbiased estimator of $V\left(\hat{t}\right)$ is

$$v\left(\hat{t}\right) = \frac{1}{n\left(n-1\right)}\sum_{i=1}^{n}\left(t_i - \hat{t}\right)^2 \tag{2.10}$$

In addition, the estimator of the population mean or total based on this sampling procedure of the selection would always have variance less than or equal to the variance of the Hansen-Hurwitz estimator used by sampling with replacement (Raj, 1956 quoted in Dryver and Thompson, 2007: 37).

Although an initial unit of size n is selected at random and without replacement, more than one of the initial sampled units may fall in the same network. Thus any network can be selected more than once. The modification of the sampling scheme of the selection for adaptive cluster sampling was introduced by Salehi and Seber (1997: 209-219) and it is called “sampling without replacement of networks”. The sampling procedure extended from sampling without replacement of networks is given by Dryver and Thompson (2007: 35-43) and is called “sampling without replacement of clusters.”

2.6 Sampling Without Replacement Networks and Clusters

2.6.1 Sampling Without Replacement of Networks

In a sampling without replacement of n networks, the first initial unit is selected by simple random sampling from the population. Then the associated network of this initial unit is added to the sample and is removed from the population before the second selection is performed. Next, the second initial unit is also selected by simple random sampling without replacement from the remaining units by ignoring the previous networks selected. The network of the second initial unit is added to the sample and will be removed from the population before the next draw is performed. This procedure will be continued until networks are selected.

Let N be the number of units in the population, n be the initial sampled size, i be the unit obtained from the i selection, m_i be the number of units in the network associated with unit i, and $y_{i.}$ be the total values of network associated with unit i.

Let $p_i = m_i/N$ be the first draw probabilities of the network including unit i and the conditional probability of selecting network containing unit i^{th} of the initial sample given all networks containing the first $(i-1)$ units in the initial sample selected be $p_i/1-p_{i-1}$, for $i = 2,3,...,n$.

An unbiased estimator of the population total based on a modified Raj type estimator (Salehi and Seber, 1997: 210-211) is

$$\hat{t}_{net} = \frac{1}{n}\sum_{i=1}^{n} t_i \quad , \tag{2.11}$$

where $t_1 = \dfrac{y_{1.}}{p_1}$ for $i=1$ and for $i = 2,3,...,n$ let $t_i = \displaystyle\sum_{j=1}^{i-1} y_{j.} + \frac{\left(1-\sum_{j=1}^{i-1} p_j\right) y_{i.}}{p_i}$.

The variance of $\hat{t}$ (Salehi and Seber, 1997: 210-211) is

$$V\left(\hat{t}\right)=\frac{1}{n^2}\sum_{i=1}^{n}V\left(t_i\right) \tag{2.12}$$

An unbiased estimator of $V\left(\hat{t}\right)$ is

$$v\left(\hat{t}\right)=\frac{1}{n\left(n-1\right)}\sum_{i=1}^{n}\left(t_i-\hat{t}\right)^2 \tag{2.13}$$

Under the adaptive design, the previous procedure was modified and was proposed by Dryver and Thompson (2007: 35-43); that is, sampling without replacement of clusters. Its detail has been represented as below.

2.6.2 Sampling Without Replacement of Clusters

In sampling of n clusters without replacement, the first initial unit is selected at random. The associated cluster is created and is added to the sample. The cluster will be removed before the next draw. Next, the second initial unit is also selected at random from the population by excluding all units in the previous observed clusters. This procedure is continued and is completed when n clusters are selected. Under this procedure, units are generally selected only once, except when a unit in the initial sample is added as an edge unit of a network or appears as an edge unit of more than one network. The pertinent probabilities p_i are not changed when the selection of the design is without replacement of clusters, but the t_i is $t_i^*=\frac{y_{1.}}{p_1}$, for $i=1$ and, for $i=2,3,\ldots,n$ let

$$t_i^*=\sum_{j\in S_{i-1}}y_{j.}+\frac{\left(1-\sum_{j\in S_{i-1}}p_j\right)y_{i.}}{p_i}\ , \tag{2.14}$$

where S_{i-1} is the collection of all distinct units obtained in the first $i-1$ draws.

An unbiased estimator of the population mean (Dryver and Thompson, 2007: 38) is

$$\hat{\mu}_{clust} = \frac{1}{Nn}\sum_{i=1}^{n} t_i^* \tag{2.15}$$

The variance of $\hat{\mu}_{clust}$ (Dryver and Thompson, 2007: 38) is

$$V\left(\hat{\mu}_{clust}\right) = \frac{1}{N^2 n^2}\sum_{i=1}^{n} V\left(t_i^*\right) \tag{2.16}$$

An unbiased estimator of $V\left(\hat{\mu}_{clust}\right)$ (Dryver and Thompson, 2007: 38) is

$$\hat{v}\left(\hat{\mu}_{clust}\right) = \frac{1}{n\left(n-1\right)}\sum_{i=1}^{n}\left(\frac{t_i^*}{N} - \hat{t}\right)^2 \tag{2.17}$$

2.6.3 The Improvement of Estimator of the Population Mean by Using a Modified Murthy Estimator

As can be seen, an unbiased estimator of the population mean when a network is selected without replacement depends on the order of the sample. The improvement of this estimator is performed by using the process of unordered sample and is shown in Salehi and Seber (1997: 209-219). The estimator based on this process is called an unordered estimator or a modified Murthy type estimator. In the study of Salehi and Seber (1997: 209-219), it is shown that the unordered estimator is an unbiased estimator of the population total and it can be rewritten in reduced form.

Let s' be an ordered sample of n networks selected without replacement, S_0 be the set of all samples obtained by permuting of the elements of s', s be the unordered sample set of networks, $P\left(s\right)$ be the probability of choosing s, and

$P(s|i)$ be the conditional probability of sample s chosen given that the network associated unit i has been chosen as the first network.

An unbiased estimator of the population total (Salehi and Seber, 1997: 211-212) is

$$\hat{t}_M = \sum_{s' \in S_0} P(s')\hat{t}_{Raj}(s') \Big/ \sum_{s' \in S_0} P(s') \tag{2.18}$$

and the reduced form of the above estimator (Salehi and Seber; 1997: 211-212) is

$$\hat{t}_M = \sum_{i=1}^{n} \frac{P(s|i)\, y_{i.}}{P(s)} \tag{2.19}$$

The variance of $\hat{t}_M$ (Salehi and Seber, 1997: 211-212) is

$$V(\hat{t}_M) = \sum_{i=1}^{K}\sum_{j<i}^{K} m_i m_j \left[1 - \sum_{s \ni i,j} P(s|i)P(s|j) \big/ P(s)\right](w_i - w_j)^2 \quad , \tag{2.20}$$

where K denotes the number of networks in the population, $w_i = y_{i.}/m_i$ is the average value of units in the network associated with unit i , and let $P(s|i,j)$ be the conditional probability of sample s given that the networks associated with unit i and j have been selected.

An unbiased estimator of $V(\hat{t}_M)$ (Salehi and Seber, 1997: 211-212) is

$$v(\hat{t}_M) = \frac{1}{(P(s))^2}\left[\sum_{i=1}^{n}\sum_{i<j}^{n} m_i m_j (w_i - w_j)^2 \{P(s)P(s|i,j) - P(s|i)P(s|j)\}\right] \tag{2.21}$$

For sampling without replacement of clusters, an unbiased estimator based on the modified Raj type of estimator was improved by using a modified Murthy type estimator and the Rao-Blackwell method (Dryver and Thompson, 2007: 38-40). The sufficient statistic, d^*, is determined as units in the sample, and their associated y-values and the corresponding networks, an indicator variable

$$e_i = \begin{cases} 1, & \text{if unit } i \text{ is selected in the initial sample} \\ 0, & \text{otherwise.} \end{cases}$$

and $d^* = \{(i, e_i, y_i) : i \in s\}$.

Note that the final sample and every value of the statistics d^* are determined by the initial sample. Define an indicator variable

$$I(s_0', d^*) = \begin{cases} 1, & \text{if } g(s_0') = d^* \\ 0, & \text{otherwise.} \end{cases},$$

where s_0 represent an ordered sample of n networks, $g(s_0')$ is the function mapping of an initial sample into a value of d^* ensuing from its selection. Under this improvement, an unbiased estimator of the population mean (Dryver and Thompson, 2007: 39-40) is

$$\hat{\mu}_{MClust} = \sum_{s_0 \in S} P(s_0) I(s_0', d^*) \hat{\mu}(s_0) \Big/ \sum_{s_0 \in S} P(s_0) I(s_0', d^*) = E\left[\hat{\mu}_{Clust} \middle| d^*\right] \quad (2.22)$$

The variance of $\hat{\mu}_{MClust}$ (based on Dryver and Thompson, 2007: 39-40) is

$$V(\hat{\mu}_{MClust}) = \frac{1}{N^2 n^2} \sum_{i=1}^{n} V(t_i^*) - \sum_{s_0 \in S} P(s_0) \left[\hat{\mu}_{Clust}(s_0) - \hat{\mu}_{MClust}(s_0)\right]^2 \quad (2.23)$$

An unbiased estimator $V(\hat{\mu}_{MClust})$ (based on Dryver and Thompson, 2007: 39-40) is

$$\hat{v}(\hat{\mu}_{MClust}) = \frac{1}{n(n-1)} \sum_{i=1}^{n} \left(\frac{t_i^*}{N} - \hat{\mu}_{Clust} \right)^2 - \frac{1}{\sum_{s_0 \in S} P(s_0) I(s_0', d^*)} \sum_{s_0 \in S} P(s_0) I(s_0', d^*) \left[\hat{\mu}_{Clust}(s_0) - \hat{\mu}_{MClust}\right]^2 \quad (2.24)$$

CHAPTER 3

ADAPTIVE CLUSTER SAMPLING WITH A SINGLE PRIMARY SAMPLING UNIT

3.1 Introduction

Although unbiased estimators of the population mean and their variances for adaptive cluster sampling design with primary and secondary sampling units (Thompson, 1991: 1103-1115) are available, all of the variance estimators require an initial sample of at least two primary sampling units (PSUs). This requirement is sometimes is too difficult to apply in a real situation, for example, when the study area is large or the study area is composed of a large number of units within a primary sampling unit. The problem of variance estimation in case of choosing a single PSU is likely to occur with conventional sampling when a systematic sample is selected. Two of eight methods of estimation which are reviewed and compared in Wolter (1984: 781-790) are based on splitting the sample into sub-samples and treating the sample as a stratified random sample. These two methods of estimation are to be used as a guide for variance estimation when a primary sampling unit is selected.

An unbiased estimator of the population mean is focused on a modified Hansen-Hurwitz type estimator which proposed by Thompson in 1991. The variance estimators proposed based on the above two methods are given in this chapter. Their properties are investigated based on simulation study. The simulation study has been carried out with real blue-winged teal data given by Smith et al. (1995: 777-778).

In the simulation, a PSU is defined as a set of secondary sampling units being arranged in strip pattern.

3.2 Adaptive Cluster Sampling with a Single Primary Sampling Unit

In the finite rare and cluster population of size $NM = qnM$ secondary sampling units (SSUs), the population consists of N primary sampling units (PSUs) with each of size M secondary sampling units (SSUs), and n is the number of primary sampling units sampled which corresponding to the number of systematic samples. When $n = 1$ primary sampling unit is chosen, possible $q = N$ primary sampling units can be selected from the population. Let Y_{ij} be the y-value of the j^{th} secondary sampling unit in the i^{th} primary sampling unit, where $i = 1, 2, ..., N$ and $j = 1, 2, ..., M$. Suppose the i^{th} primary sampling unit is selected with probabilities $1/N$, let $y_j^* = Y_{ij}$, where $j = 1, 2, ..., M$, y_j be the total y-value of secondary sampling units within the network associated with y_j^* , x_j be the number of primary sampling units in the network associated with y_j , and $y_j = y_k$ and $x_j = x_k$. The draw-by-draw probability of the selection of PSU associated with network of y_j^* is x_j / N .

In adaptive cluster sampling (ACS) with a single primary sampling unit chosen, an unbiased estimator of the population mean proposed by Thompson (1992: 293-294) is

$$\hat{\mu} = \hat{\mu}_i = \frac{1}{M} \sum_{k \in \kappa_i} \frac{y_k}{x_k} \tag{3.1}$$

with the variance

$$V\left(\hat{\mu}\right)=V\left(\hat{\mu}_i\right)=\left(\frac{1}{N}\right)\sum_{i=1}^{N}\left(\hat{\mu}_i-\mu\right)^2 , \qquad (3.2)$$

where $\hat{\mu}_i$ represents the sample mean if the i^{th} primary sampling unit is selected and κ_i is the number of networks associated with the selected primary sampling unit.

3.2.1 Methodology and Estimators of the Variance

In order to find the estimator of $V\left(\hat{\mu}\right)$ in equation 3.2, the principle of two methods of estimation in conventional systematic sampling (Cochran, 1977: 223-226, Wolter, 1984: 781-790) is used as a guide for estimation.

Method I

In method I, the selected primary sampling unit is divided into $p = M/m$ sub-samples, where $p \geq 2$. Each sub-sample contains m secondary sampling units and $M = \sum_{t=1}^{p} m = mp$. Let $m \geq 2$ secondary sampling units be assigned to each sub-sample at random, $\hat{\mu}_t = \frac{1}{m}\sum_{k\in\kappa_t}\frac{y_k}{h_k x_k}$ be the mean of the t^{th} sub-sample, t represent the sub-samples, h_k be the number of sub-samples in which the k^{th} network appears, κ_t be the number of networks associated with the t^{th} sub-sample and $t = 1, 2, .., p$ and $p_i = 1/N$ be the probability of the i^{th} PSU chosen. An unbiased estimator of the population mean in equation 3.1 can be rewritten in terms of $\hat{\mu}_t$, as

$$\hat{\mu} = \hat{\mu}_i = \frac{1}{p}\sum_{t=1}^{p}\hat{\mu}_{t(i)} \quad , \qquad (3.3)$$

where i represent the i^{th} primary sampling unit that is chosen.

Since $\hat{\mu}$ is the mean of $\hat{\mu}_t$ for simple random sample of size p, the formulae for the variance can be rewritten as

$$V\left(\hat{\mu}\right) = V\left(\hat{\mu}_i\right) = \frac{1}{Np^2}\sum_{i=1}^{N}\left\{\left(\sum_{t=1}^{p}\hat{\mu}_{t(i)}\right) - p\mu\right\}^2 \tag{3.4}$$

The derivation of the estimator of $V\left(\hat{\mu}\right)$ is obtained by replacing z_t instead of $\hat{\mu}_t$, the variance estimator based on method I is of the form

$$\hat{v}_{acs(1)}\left(\hat{\mu}_i\right) = \hat{v}_{acs(1)}\left(\hat{\mu}\right) = v\left[\frac{1}{p}\sum_{t=1}^{p}\hat{\mu}_t\right] = \frac{1}{p^2}\sum_{t=1}^{p}v\left(z_t\right)$$

$$= \frac{\left(1-f\right)}{p}\sum_{t=1}^{p}\frac{\left(z_t - \overline{z}\right)^2}{p-1}$$

$$= \frac{\left(1-f\right)}{p\left(p-1\right)}\sum_{t=1}^{p}\left(\hat{\mu}_t - \hat{\mu}\right)^2 \tag{3.5}$$

Theorem 3.1 If the initial sample of size $M = mp$ is divided at random into p sub-samples and $m = \frac{M}{p}$ secondary sampling units are assigned to each sub-sample at random then a biased estimator of $V(\hat{\mu})$ (based on Wolter, 1984: 781-790) is

$$\hat{v}_{acs(1)}\left(\hat{\mu}\right) = \left(1-f\right)\frac{1}{p}\frac{\sum_{t=1}^{p}\left(\hat{\mu}_t - \hat{\mu}\right)^2}{p-1} \quad , \tag{3.6}$$

where $f = 1/N$ and $\hat{\mu}_t$ is the mean of the t^{th} sub-sample, $\hat{\mu}$ is as in equation 3.3.

Method II

For method II, the selected primary sampling unit is regarding as a stratified sample with $p = M/2$ strata of equal size, where $p \geq 2$. Assume that M is an even integer and no prior information about population. A reasonable choice for allocated units into stratum would be to assign equal sample size to the stratum (Thompson, 1992: 107). Define t represents the t^{th} stratum, where $t = 1, 2, ..., p$. So that for stratum t the sample size would be $m_t = M/(M/2) = 2$ secondary sampling units. Let $w_t = m_t/M$ be the weight of stratum t and $\hat{\mu}_t' = \sum_{j=1}^{m=2} u_{jt} \big/ 2$, where $u_{jt} = \sum_{k \in \kappa_{jt}} \frac{y_k}{v_k x_k}$. Let κ_{jt} be the number of networks associated with the j^{th} secondary sampling unit within stratum t and v_k be the number of times that the k^{th} network appears. So the unbiased estimator of the population mean in equation 3.1 can be rewritten of the form

$$\hat{\mu} = \hat{\mu}_i = \sum_{t=1}^{p=M/2} w_t \hat{\mu}_t' = \frac{1}{M/2} \sum_{t=1}^{p=M/2} \hat{\mu}_t' \tag{3.7}$$

with the variance

$$V(\hat{\mu}) = \frac{1}{N} \sum_{i=1}^{N} \left[\left(\sum_{t=1}^{p} w_t \hat{\mu}_{it}' - p\mu \right)^2 \right] \tag{3.8}$$

The second variance estimator $\hat{v}_{acs(2)}(\hat{\mu})$ can be obtained as follows:

$$\hat{v}_{acs(2)}(\hat{\mu}_i) = \hat{v}_{acs(2)}(\hat{\mu}) = \hat{v}\left[\left(\sum_{t=1}^{p=M/2} w_t \hat{\mu}_t \right) \right] = \left(\sum_{t=1}^{p=M/2} w_t^2 \hat{v}(\hat{\mu}_t) \right)$$

$$= \left(\sum_{t=1}^{p} w_t^2 (1 - f_t) \frac{s_t^2}{m_t} \right) = \left(\sum_{t=1}^{p} w_t^2 (1 - f_t) \frac{\sum_{j \in t} (u_{jt} - \hat{\mu}_t)^2}{m_t (m_t - 1)} \right)$$

$$= \left(\sum_{t=1}^{p} w_t^2 \left(1 - f_t\right) \frac{\sum_{j \in t} \left(u_{jt} - \left(\left(u_{1t} + u_{2t}\right)/2\right)\right)^2}{m_t \left(m_t - 1\right)} \right)$$

$$\hat{v}_{acs(2)}\left(\hat{\mu}\right) = \left(\sum_{t=1}^{p} \left(m_t / M\right)^2 \left(1 - f_t\right) \left(\frac{2\left(u_{1t} - u_{2t}\right)^2}{4m_t\left(m_t - 1\right)} \right) \right)$$

A value of f_t is not known so that we approximate the value of f_t with $f = M/NM$. Since each stratum contains $m_t = 2$ secondary sampling units so we get

$$\hat{v}_{acs(2)}\left(\hat{\mu}\right) = \left(\sum_{t=1}^{p} \left(2/M\right)^2 \left(1 - \frac{1}{N}\right) \left(\frac{\left(u_{1t} - u_{2t}\right)^2}{4} \right) \right)$$

$$= \left(\left(1 - f\right)\left(4/M^2\right) \sum_{t=1}^{p} \frac{\left(u_{1t} - u_{2t}\right)^2}{4} \right)$$

$$= \left(\left(\frac{1 - f}{M}\right) \sum_{t=1}^{p} \frac{\left(u_{1t} - u_{2t}\right)^2}{M} \right) \qquad (3.9)$$

Theorem 3.2 If the initial sample of size $M = mp$ is treated as if it were $p = \frac{M}{2}$ strata with each stratum containing equal size and $m_t = 2$ secondary sampling units are assigned to each stratum at random, then a biased estimator of $V\left(\hat{\mu}\right)$ (based on Cochran, 1977: 226-227; Wolter, 1984: : 781-790) is

$$\hat{v}_{acs(2)}\left(\hat{\mu}\right) = \left(1 - f\right) \frac{1}{M} \sum_{t=1}^{p} \frac{\left(u_{2t} - u_{1t}\right)^2}{M} \qquad (3.10)$$

where $f = 1/N$, $\hat{\mu}$ is as in equation 3.3.

An example of the bias of variance estimators in equation 3.6 and 3.10 has been shown with a small population in section 3.2.2.

3.2.2 An Illustrative Example

An artificial population area of 24 meter-squares was divided into $NM = 24$ secondary sampling units or plots. Each secondary sampling unit is of size 1 meter-squares. Table 3.1 shows the details of the individual value of population, Y_{ij}, and the corresponding value of the network, y_k, which is sometimes called the transformed population. This population with giving the population mean of μ = 3.33 and population variance of σ^2 = 29.19 is partitioned into $N = 6$ primary sampling units (PSUs). A primary sampling unit is defined as the set of secondary sampling units (SSUs) being arranged in horizontal strip. The number of primary sampling units associated with the transformed population, x_k, is shown in table 3.1. The condition for adding units is defined by $C = \{Y_{ij} \geq 5\}$. In the case of $N = 6$ PSUs, a single random start is selected from 6x1 squares in the upper left of table 3.1. All units with the same position are formed the selected primary sampling unit.

Table 3.1 A Population Consists of $N = 6$ PSUs and Its Associated Network with the Number of PSUs Corresponding with the Network.

Population; Y_{ij}				⇨	Corresponding Network Value; y_k *				⇨	# of PSUs Associated with the Network; x_k			
9	0	4	0		9	0	4	0		1	1	1	1
0	0	1	0		0	0	1	0		1	1	1	1
1	0	0	8		1	0	0	8+20+15+6+10 = 59		1	1	1	3
0	6	15	20		0	59	59	59		1	3	3	3
3	1	0	10		3	1	0	59		1	1	1	3
0	2	0	0		0	2	0	0		1	1	1	1

Three properties such that the relative bias (RB), mean squared error (MSE), and the relative frequency of coverage μ of the 95% confidence interval that is formed by each variance estimator are considered. Moreover, the efficiency of the proposed variance estimators for adaptive cluster sampling (ACS) with a single PSU is compared to non-adaptive variance estimators in conventional systematic sampling (SYS). Let subscript $(\bullet)$ be replaced with "acs(1)", "acs(2)", "sys(1) and "sys(2)" for the variance estimators in ACS, and SYS, respectively. Let subscript i denote the selected PSU, $p_i = 1/N$ be the probability of chosen the i^{th} PSU. In each case of N, total $r = \frac{M!}{(m!)^p\ p!}$ possible ways for assign units to sub- samples were performed and all possible choices of choosing a single PSU were considered. Define s represents the possible sample obtained in each case N and p. For the s^{th} sample, define $u_{s(\bullet)}$ to be 1 if the 95% confidence interval that is formed by using $\hat{\mu}_{(\bullet)} \pm t_{\alpha/2,(p-1)}\sqrt{\hat{v}_{(\bullet)}(\hat{\mu})}$ contains μ, and 0 if it does not. The 95% confidence interval is done by assuming that the population is normal. The relative frequency of coverage μ of 95% CI that is formed by using each estimator of variance is calculated by $\sum_s u_{s(\bullet)} p(s)$.

Based on information in table 3.1 it will be shown how to calculate estimators of the population mean and variance estimators for ACS and SYS. Suppose that the 5th primary sampling unit (PSU) with giving an individual y-value {3, 1, 0, 10} and network value {3, 1, 0, 59} is selected as the initial sample. This initial sample is split up at random into $p = 2$ sub-samples/ strata. Suppose the first-two secondary sampling units (SSUs) is in the 1st sub-sample/ strata and the remaining SSUs in the initial sample is in the 2nd sub-sample/ stratum. Values of y_k and x_k for units within the 1st sub-sample are {3, 1} and {1, 1} while values of y_k and x_k for units within the 2nd sub-sample are {0, 59} and {1,3}, respectively. Notice that the information of edge units is not used in this study.

Based on method I, values of h_k are {1, 1} and {1, 1} for units in the 1st sub-sample and the 2nd sub-sample, respectively. Thus we get

$$\hat{\mu}_1 = \frac{1}{2}\left(\frac{3}{1(1)} + \frac{1}{1(1)}\right) = 2 \text{ and } \hat{\mu}_2 = \frac{1}{2}\left(\frac{0}{1(1)} + \frac{59}{1(3)}\right) = 9.83.$$

Estimators $\hat{\mu} = \hat{\mu}_{acs(1)}$ and $\hat{v}_{acs(1)}(\hat{\mu})$ are calculated by

$$\hat{\mu}_{acs(1)} = \frac{1}{2}(2 + 9.83) = 5.92, \tag{3.11}$$

and

$$\hat{v}_{acs(1)}(\hat{\mu}) = \left(\frac{6-1}{6}\right)\frac{1}{2(2-1)}\left((2-5.92)^2 + (9.83-5.92)^2\right) = 12.77 \tag{3.12}$$

For non-adaptive estimators, we get $\bar{y}_1 = \left(\frac{3+1}{2}\right) = 2$ and $\bar{y}_2 = \left(\frac{0+10}{2}\right) = 5$.

Estimators $\bar{y}_{sys}$ and $\hat{v}_{sys(1)}(\bar{y})$ are calculated by

$$\bar{y}_{sys} = \frac{1}{2}(2+5) = 3.50, \tag{3.13}$$

and

$$\hat{v}_{sys(1)}(\bar{y}) = \left(\frac{6-1}{6}\right)\frac{1}{2(2-1)}\left((2-3.50)^2 + (5-3.50)^2\right) = 1.88 \tag{3.14}$$

Based on method II, values of v_k is {1, 1} and {1, 1} for units in the 1st stratum and the 2nd stratum, respectively. Thus we get

$$u_{11} = \frac{3}{1(1)} = 3,\ u_{21} = \frac{1}{1(1)} = 1 \text{ and } u_{12} = \frac{0}{1(1)} = \frac{0}{1} = 0,\ u_{22} = \frac{59}{1(3)} = 19.67,$$

$$\hat{\mu}'_1 = \frac{(3+1)}{2} = 2 \text{ and } \hat{\mu}'_2 = \frac{1}{2}\left(0 + \frac{59}{3}\right) = 9.83.$$

Estimators $\hat{\mu} = \hat{\mu}_{acs(2)}$ and $\hat{v}_{acs(2)}(\hat{\mu})$ are calculated by

$$\hat{\mu}_{acs(2)} = \frac{1}{2}(2 + 9.83) = 5.92 \tag{3.15}$$

and

$$\hat{v}_{acs(2)}(\hat{\mu}) = \left(\frac{6-1}{6}\right)\frac{1}{4^2}\left((3-1)^2 + \left(0 - \frac{59}{3}\right)^2\right) = 20.35 \tag{3.16}$$

For non-adaptive estimators, we get $y_1 = 3$, $y_2 = 1$ for units in the 1st stratum and $y_1 = 0$, $y_2 = 10$ for units in the 2nd stratum. Thus an estimator $\hat{v}_{sys(2)}(\bar{y})$ is calculated by

$$\hat{v}_{sys(2)}(\bar{y}) = \left(\frac{6-1}{6}\right)\frac{1}{4^2}\left((3-1)^2 + (0-10)^2\right) = 5.42 \tag{3.17}$$

For the population consists of $N = 6$ PSUs, since the initial sample contains $M = 4$ SSUs so both methods of estimation can be obtained in the case of $p = 2$ sub-samples/ strata. Total 3 possible ways for assign units to sub-samples are performed here. The list of 18 possible samples is shown in table 3.3. The 95% confidence interval of μ is also calculated from all possible cases by assuming that the population is normal and the result is represented in table 3.4. Six possible sample means are shown in table 3.2. Its expectation is 3.33 and the actual variance is 5.01.

When the population consists of $N = 6$ PSUs and $p = 2$ sub-samples/ strata, the result in table 3.3 shows that both estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ are overestimated. The estimator $\hat{v}_{acs(1)}(\hat{\mu})$ is better than estimator $\hat{v}_{acs(2)}(\hat{\mu})$ in terms of RB because the RB of the first estimator is smaller than the second. When these two variance estimators are compared to each other in terms of MSE, the MSE of estimator $\hat{v}_{acs(2)}(\hat{\mu})$ is smaller than the MSE of estimator $\hat{v}_{acs(1)}(\hat{\mu})$. The relative frequency of coverage μ of 95% CI that is formed by estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ shows the ability to construct 95% CI of the second estimator is higher 16.6 percent than the first (see table 3.4). When both estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ are compared to non-adaptive estimators $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$, the results in table 3.3 show that the two new variance estimators are better than non-adaptive estimators $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ in terms of minimum relative bias. As seen in table 3.4, the ability to construct CI of estimators $\hat{v}_{sys(2)}(\bar{y})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ is same because the relative frequency of coverage μ of 95% CI that is formed with these two estimators is 0.833 or 83.3 percent. But the relative frequency of coverage μ of 95% CI that is formed with estimators $\hat{v}_{acs(2)}(\hat{\mu})$ is higher 5.5 percent than the relative frequency of overage μ of 95% CI that is formed with $\hat{v}_{sys(1)}(\bar{y})$. In addition, the ability to construct CI containing μ of estimator $\hat{v}_{acs(1)}(\hat{\mu})$ is lowest, that is 0.667 or 66.7 percent. However, the ability to construct CI of all estimators is less than 95 percent.

Table 3.2 All Possible Samples When a Single PSU is Selected and the Corresponding Estimators $\hat{\mu}_{acs(1)}$, $\hat{\mu}_{acs(2)}$, $\bar{y}_{sys(1)}$ and $\bar{y}_{sys(2)}$.

N	**Observed unit ***	$P(s)$	**ACS**		**SYS**	
			$\hat{\mu}_{acs(1)}$	$\hat{\mu}_{acs(2)}$	$\bar{y}_{sys(1)}$	$\bar{y}_{sys(2)}$
	(9, 0, 4, 0; 0)	1/6	3.15	3.25	3.25	3.25
	(0, 0, 1, 0)	1/6	0.25	0.25	0.25	0.25
6	(1, 0, 0, 8; 20, 15, 6, 10, 0, 0, 0, 0, 0, 0, 1)	1/6	5.17	5.17	2.25	2.25
	(0, 6, 15, 20; 8, 10, 0, 0, 0, 0, 0, 0, 1)	1/6	4.92	4.92	10.25	10.25
	(3, 1, 0, 10; 20, 15, 6, 8, 0, 0, 0, 0, 0)	1/6	5.92	5.92	3.50	3.50
	(0, 2, 0, 0)	1/6	0.50	0.50	0.50	0.50
Mean		-	**3.33**	**3.33**	**3.33**	**3.33**
Variance		-	**5.01**	**5.01**	**11.09**	**11.09**

Note: ***** refers to the selected PSU. The additional units is representing after the semicolon in the parenthesis and $P(s) = p_i$.

Table 3.3 All Possible Samples When $N=6$ PSUs, $p=2$ and the Corresponding Estimators $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$.

Observed unit *	Units in sub-sample / stratum	$P(s)$ **	ACS		SYS	
			$\hat{v}_{acs(1)}(\hat{\mu})$	$\hat{v}_{acs(2)}(\hat{\mu})$	$\hat{v}_{sys(1)}(\bar{y})$	$\hat{v}_{sys(2)}(\bar{y})$
(9, 0, 4, 0; 0)	(9, 0) (4, 0)	1/18	1.30	5.05	1.30	5.05
	(9, 4) (0, 0)	1/18	8.80	1.30	8.80	1.30
	(9, 0) (0, 4)	1/18	1.30	5.05	1.30	5.05
(0, 0, 1, 0)	(0, 0) (1, 0)	1/18	0.05	0.05	0.05	0.05
	(0, 1) (0, 0)	1/18	0.05	0.05	0.05	0.05
	(0, 0) (0, 1)	1/18	0.05	0.05	0.05	0.05
(1, 0, 0, 8; 20, 15, 6, 10, 0, 0, 0, 0, 0, 1)	(1, 0) (0, 8)	1/18	18.15	20.19	2.55	3.39
	(1, 0) (0, 8)	1/18	18.15	20.19	2.55	3.39
	(1, 8) (0, 0)	1/18	22.25	18.15	4.22	2.55
(0, 6, 15, 20; 8, 10, 0, 0, 0, 0, 0, 1)	(0, 6) (15, 20)	1/18	0.00	2.24	43.80	3.18
	(0, 15) (6, 20)	1/18	0.00	2.24	6.30	21.93
	(0, 20) (6, 15)	1/18	0.00	2.24	0.05	25.05
(3, 1, 0, 10; 20, 15, 6, 8, 0, 0, 0, 0)	(3, 1) (0, 10)	1/18	12.78	20.35	1.88	5.42
	(3, 0) (1, 10)	1/18	16.26	18.62	3.33	4.69
	(3, 10) (1, 0)	1/18	24.45	14.52	7.50	2.60
(0, 2, 0, 0)	(0, 2) (0, 0)	1/18	0.21	0.21	0.21	0.21
	(0, 0) (2, 0)	1/18	0.21	0.21	0.21	0.21
	(0, 0) (2, 0)	1/18	0.21	0.21	0.21	0.21
Mean			**6.90**	**7.27**	**4.69**	**4.69**
Relative bias			**0.376**	**0.451**	**-0.578**	**-0.578**
MSE			**81.49**	**73.57**	**138.06**	**89.05**

Note: * refers to the selected PSU, ** $P(s)=p_i \times p(subsample)$. The additional units is representing after the semicolon in the parenthesis.

Table 3.4 All Possible Samples When $N = 6$ PSUs, $p = 2$, the 95% CI of μ That is Formed by Using $\hat{\mu}_{(\bullet)} \pm t_{\alpha/2, df} \sqrt{\hat{v}_{(\bullet)}(\hat{\mu})}$ and the Relative Frequency of Coverage μ of the 95% CI.

Observed unit *	Units in sub-sample / stratum	ACS		SYS	
		$\hat{v}_{acs(1)}(\hat{\mu})$	$\hat{v}_{acs(2)}(\hat{\mu})$	$\hat{v}_{sys(1)}(\bar{y})$	$\hat{v}_{sys(2)}(\bar{y})$
(9, 0, 4, 0; 0)	(9, 0) (4, 0)	(-11.24, 17.74)	(-25.30, 31.80)	(-11.24, 17.74)	(-25.30, 31.80)
	(9, 4) (0, 0)	(-34.44, 40.94)	(-11.24, 17.74)	(-34.44, 40.94)	(-11.24, 17.74)
	(9, 0) (0, 4)	(-11.24, 17.74)	(-25.30, 31.80)	(-11.24, 17.74)	(-25.30, 31.80)
(0, 0, 1, 0)	(0, 0) (1, 0)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)
	(0, 1) (0, 0)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)
	(0, 0) (0, 1)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)	(-2.64, 3.14)
(1, 0, 0, 8; 20, 15, 6, 10, 0, 0, 0, 0, 0, 0, 1)	(1, 0) (0, 8)	(-48.96, 59.29)	(-51.93, 62.26)	(-18.04, 22.54)	(-21.12, 25.62)
	(1, 0) (0, 8)	(-48.96, 59.29)	(-51.93, 62.26)	(-18.04, 22.54)	(-21.12, 25.62)
	(1, 8) (0, 0)	(-54.76, 65.09)	(-48.96, 59.29)	(-23.84, 28.34)	(-18.04, 22.54)
(0, 6, 15, 20; 8, 10, 0, 0, 0, 0, 0, 0, 1)	(0, 6) (15, 20)	(4.91, 4.91)	(-14.09, 23.92)	(-73.84, 94.34)	(-12.39, 32.89)
	(0, 15) (6, 20)	(4.91, 4.91)	(-14.09, 23.92)	(-21.64, 42.14)	(-49.24, 69.74)
	(0, 20) (6, 15)	(4.91, 4.91)	(-14.09, 23.92)	(7.35, 13.14)	(-53.34, 73.84)
(3, 1, 0, 10; 20, 15, 6, 8, 0, 0, 0, 0, 0)	(3, 1) (0, 10)	(-39.51, 51.34)	(-51.40, 63.23)	(-13.89, 20.89)	(-26.07, 33.07)
	(3, 0) (1, 10)	(-45.31, 57.14)	(-48.90, 60.74)	(-19.69, 26.69)	(-24.00, 31.00)
	(3, 10) (1, 0)	(-56.91, 68.74)	(-42.49, 54.33)	(-31.29, 38.29)	(-17.00, 24.00)
(0, 2, 0, 0)	(0, 2) (0, 0)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)
	(0, 0) (2, 0)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)
	(0, 0) (2, 0)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)	(-5.29, 6.29)
The relative frequency of coverage μ		**0.667**	**0.833**	**0.778**	**0.833**

Note: * refers to the selected PSU. The additional units is representing after the semicolon in the parenthesis.

3.3 Simulation Study

As seen in section 3.2, the formulae of variance estimators proposed for ACS where a single PSU is drawn is too complicated to make a mathematics comparison. In this section, properties and efficiency comparison of proposed variance estimators has been done by using simulation. The properties investigation has been carried out with a real blue-winged teal given by Smith et al. (1995: 777-778). For real blue-winged teal data, a primary sampling unit is defined only as a set of second sampling units being arranged in strip pattern.

3.3.1 Simulation Study

In this part of the study, a simulation has been carried out with real blue winged-teal data which was studied in the region of 5,000 kilometer-squares in Central Florida and was given by Smith et al. (1995: 777-778). This data is partitioned into 200 secondary sampling units (SSUs) with each of size 25 kilometer-squares. The details are shown in figure 3.1, and the number that appears in cell $i-j$ is the value of Y_{ij}. The population mean and population variance are μ = 70.605 and σ^2 = 451,440.97. The pre-condition for adding an unit is defined by $C = \{Y_{ij} \geq 1\}$.

0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	20	4	2	12	0	0	0	0	0	10	103	0	0	0
0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	150	7144	1	0
0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0	6	6339	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14	122	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	114	60
0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	3	0

Figure 3.1 Real Blue-Winged Teal Data with μ =70.605 and σ^2 =451,440.97.

These data are considered here in the two cases of $N = 10$ and $N = 5$ primary sampling units (PSUs) in the population of size $M = 20$ and $M = 40$ secondary sampling units (SSUs), respectively. Since a PSU is defined as a set of SSUs that are arranged in strip pattern in simulation section so a single random start is chosen at random from 10x1 squares and 5x1 squares in the upper left corner of figure 3.1 for $N = 10$ and $N = 5$ PSUs, respectively. The positions repeated throughout the study area and a set of these SSUs form the selected PSU. All possible choices of initial samples obtained from the selection of a single PSU are performed. Both methods of estimation of variance are carried out using the Visual-Basic package and replicated $r = 2,000$ times. The aim of simulation study is to make a comparison between proposed estimators of the variance obtained from method I and method II and non-adaptive variance estimators in terms of relative bias (RB), mean squared error (MSE) and the relative frequency of coverage μ of 95% confidence intervals that is produced by using each estimator of variance. So that let subscript $(\bullet)$ be defined as in section 3.2.2 and let subscript j represents the number of iterations.

The simulation mean and variance of $\hat{\mu}_{(\bullet)}$ is done based on the formula

$$E\left(\hat{\mu}_{(\bullet)}\right) = \bar{\mu}_{(\bullet)} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{r}\hat{\mu}_{(\bullet)ij}}{rN}, \tag{3.18}$$

where i denote the i^{th} PSU that is selected, and

$$v\left(\hat{\mu}_{(\bullet)}\right) = \frac{1}{N(r-1)}\sum_{i=1}^{N}\sum_{j=1}^{r}\left(\hat{\mu}_{(\bullet)ij} - \mu\right)^2, \tag{3.19}$$

where $\bar{\mu}_{(\bullet)}$ is as in equation 3.13.

The simulation mean of $\hat{v}_{(\bullet)}(\hat{\mu})$ is done based on the formula

$$E\left(\hat{v}_{(\bullet)}(\hat{\mu})\right) = \overline{v}_{(\bullet)} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{r}\hat{v}_{(\bullet)ij}(\hat{\mu})}{rN}, \tag{3.20}$$

The relative bias of estimator $\hat{v}_{(\bullet)}(\hat{\mu})$ is calculated based on the formula

$$RB\left(\hat{v}_{(\bullet)}(\hat{\mu})\right) = \frac{E\left(\hat{v}_{(\bullet)}(\hat{\mu})\right)}{v\left(\hat{\mu}_{(\bullet)}\right)} - 1, \tag{3.21}$$

where $v\left(\hat{\mu}_{(\bullet)}\right)$ and $E\left(\hat{v}_{(\bullet)}(\hat{\mu})\right)$ are as in equation 3.14 and 3.15, respectively.

The simulation of mean squared error (MSE) of estimator $\hat{v}_{(\bullet)}(\hat{\mu})$ is done based on the formula

$$MSE\left(\hat{v}_{(\bullet)}(\hat{\mu})\right) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{r}\left[\left(\hat{v}_{(\bullet)ij}(\hat{\mu})\right) - v\left(\hat{\mu}_{(\bullet)}\right)\right]^2}{rN}, \tag{3.22}$$

where $v\left(\hat{\mu}_{(\bullet)}\right)$ is as in equation 3.14.

The confidence interval is performed based on a normality assumption so that the 95% confidence interval is calculated by

$$\hat{\mu}_{(\bullet)ij} \pm t_{\alpha/2,(p-1)}\sqrt{\hat{v}_{(\bullet)ij}(\hat{\mu})} \tag{3.23}$$

Define $u_{ij}\left(\hat{v}_{(\bullet)}(\hat{\mu})\right)$ to be 1 if the 95% confidence interval that is produced by using equation 3.18 contains μ, and 0 if the 95% confidence interval does not contain μ. So that the relative frequency of coverage μ of 95% confidence interval of is calculated base on the formula

$$CI\left(\hat{v}_{(\bullet)}\left(\hat{\mu}\right)\right)=\frac{1}{rN}\sum_{i=1}^{N}\sum_{j=1}^{r}u_{ij}\left(\hat{v}_{(\bullet)}\left(\hat{\mu}\right)\right) \qquad (3.24)$$

The results of the variance estimation when the initial sample of a single PSU is drawn are as follows:

When N = 10 PSUs and each PSU consists of M = 20 SSUs, the initial sample is split up into four possible values of p; these are p = 2, 4, 5 and 10 sub-samples/ strata. For cases of p = 2, 4 and 5, the estimator $\hat{v}_{acs(2)}\left(\hat{\mu}\right)$ and $\hat{v}_{sys(2)}\left(\bar{y}\right)$ cannot be obtained because the number of units in each stratum is not 2. This limitation occurs when N = 5 PSUs and p are 2, 4, 5, 8 and 10. In these cases, only estimators $\hat{v}_{acs(1)}\left(\hat{\mu}\right)$ and $\hat{v}_{sys(1)}\left(\bar{y}\right)$ are compared the properties in terms of relative bias (RB), MSE and the ability to construct 95% CI for μ. So that two new variance estimators are compared to each other in two cases of N = 10 PSUs, p = 10 sub-samples/ strata and N = 5 PSUs, p = 20 samples/ strata.

The simulation means and variance of estimators $\hat{\mu}_{acs}$ and $\bar{y}_{sys}$ for every p when N = 10 PSUs and N = 5 PSUs is shown in table 3.5 to table 3.6, respectively. The results show that both estimators $\hat{\mu}_{acs}$ and $\bar{y}_{sys}$ are unbiased estimators of the population mean. When considering the estimated of true variance of $\hat{\mu}_{acs}$ and $\bar{y}_{sys}$, the results in table 3.5 and 3.6 show that estimator $\hat{\mu}_{acs}$ is more efficient than estimator $\bar{y}_{sys}$ in every cases of p and N. The simulation means and MSE of estimators $\hat{v}_{acs(1)}\left(\hat{\mu}\right)$ and $\hat{v}_{sys(1)}\left(\bar{y}\right)$ when N = 10 PSUs, p = 2, 4, 5, 10 and N = 5 PSUs, p = 2, 4, 5, 8, 10 and 20 are shown in table 3.7 and table 3.8. The relative bias of these two estimators is shown in table 3.9. The results show that estimator $\hat{v}_{acs(1)}\left(\hat{\mu}\right)$ is underestimated, while estimator $\hat{v}_{sys(1)}\left(\bar{y}\right)$ is overestimated. The estimator $\hat{v}_{acs(1)}\left(\hat{\mu}\right)$ gives a large bias (RB) when $N=10$ PSUs, p = 2 sub-samples. For $N=10$ PSUs and p = 2, 4, 5 and 10, estimator $\hat{v}_{acs(1)}\left(\hat{\mu}\right)$ is though less efficient than estimator $\hat{v}_{sys(1)}\left(\bar{y}\right)$ in terms of relative bias its bias is slightly decreased when p

increases. As seen in table 3.9, the relative bias of estimator $\hat{v}_{acs(1)}(\hat{\mu})$ is 21 percent decreased when value of p is changed from 2 to 4. When considering in terms of MSE, MSE of both estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{sys(1)}(\bar{y})$ are decreased when p increased (see table 3.7). In addition, estimator $\hat{v}_{acs(1)}(\hat{\mu})$ has the smaller MSE than estimator $\hat{v}_{sys(1)}(\bar{y})$ for every p (see table 3.7). When $N = 5$ PSUs and p = 2, 4, 5, 8, 10 and 20 sub-samples, estimator $\hat{v}_{acs(1)}(\hat{\mu})$ is more efficient than estimator $\hat{v}_{sys(1)}(\bar{y})$ in terms of minimum relative bias and MSE (see table 3.8 and table 3.9).

When N =10 PSUs, p = 10 and N =5 PSUs, p = 20, estimators $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ can be obtained. When all these estimators are compared to each other, estimator $\hat{v}_{sys(1)}(\bar{y})$ seems to be a good choice in terms of minimum RB when N =10 and p = 10. But when N =5 and p = 20, estimator $\hat{v}_{acs(2)}(\hat{\mu})$ seems to be a good choice because it has the smallest RB (see table 3.9). When two new estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ are compared to each other, the results in table 3.9 show that second estimator is better than the first because it has the smaller RB for both cases of N =10 PSUs, p = 10 and N =5 PSUs, p = 20. When these two estimators are compared in terms of MSE, estimator $\hat{v}_{acs(2)}(\hat{\mu})$ gives the smallest MSE. In addition, estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ have the smaller MSE than non-adaptive variance estimators $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ for both cases of N =10 PSUs, p = 10 and N =5 PSUs, p = 20 (see table 3.7 and table 3.8).

The relative frequency of coverage μ of 95% confidence interval that is formed by $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ is shown in table 3.10. The ability to construct intervals containing μ of estimator $\hat{v}_{acs(1)}(\hat{\mu})$ increased 13.9 percent and 35.5 percent when p is changed from 2 to 4 in cases of N =10 PSUs and N =5 PSUs, respectively. However, estimators $\hat{v}_{acs(1)}(\hat{\mu})$ and $\hat{v}_{acs(2)}(\hat{\mu})$ have the same ability to construct intervals containing μ because the relative frequency of coverage

μ of 95% confidence interval that is formed by these two estimators is maximized at 30 percent and 60 percent when N =10 PSUs, p = 10 and N =5 PSUs, p = 20. The maximum relative frequency of coverage μ of 95% confidence interval that is formed by estimators $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ is 0.30 or 30 percent and 0.40 or 40 percent when N =10 PSUs, p = 10 and N =5 PSUs, p = 20. This means that the ability to construct intervals of all these variance estimators is less than 95 percent (see table 3.10).

Table 3.5 The Simulation Mean and Variance of $\hat{\mu}_{acs}$ and $\overline{y}_{sys}$ When the Real Blue-Winged Teal Population Consists of $N = 10$ PSUs.

p	**Possible PSU chosen**	$\hat{\mu}_{acs}$	$\overline{y}_{sys}$
2	1	0.25	0.25
	2	0.15	0.15
	3	0.00	0.00
	4	7.55	7.55
	5	364.90	364.90
	6	317.45	317.45
	7	0.00	0.00
	8	6.80	6.80
	9	8.70	8.70
	10	0.25	0.25
Mean		**70.605**	**70.605**
Variance		**10894.258**	**20514.016**
4	1	0.25	0.25
	2	0.15	0.15
	3	0.00	0.00
	4	7.55	7.55
	5	364.90	364.90
	6	317.45	317.45
	7	0.00	0.00
	8	6.80	6.80
	9	8.70	8.70
	10	0.25	0.25
Mean		**70.605**	**70.605**
Variance		**10894.258**	**20514.016**
5	1	0.25	0.25
	2	0.15	0.15
	3	0.00	0.00
	4	7.55	7.55
	5	364.90	364.90
	6	317.45	317.45
	7	0.00	0.00
	8	6.80	6.80
	9	8.70	8.70
	10	0.25	0.25
Mean		**70.605**	**70.605**
Variance		**10894.258**	**20514.016**
10	1	0.25	0.25
	2	0.15	0.15
	3	0.00	0.00
	4	7.55	7.55
	5	364.90	364.90
	6	317.45	317.45
	7	0.00	0.00
	8	6.80	6.80
	9	8.70	8.70
	10	0.25	0.25
Mean		**70.605**	**70.605**
Variance		**10894.258**	**20514.016**

Note: Mean refers to the simulation mean of estimators $\hat{\mu}_{acs}$ and $\overline{y}_{sys}$.

Table 3.6 The Simulation Mean and Variance of $\hat{\mu}_{acs}$ and $\bar{y}_{sys}$ When the Real Blue-Winged Teal Population Consists of $N = 5$ PSUs.

p	**Possible PSU chosen**	$\hat{\mu}_{acs}$	$\bar{y}_{sys}$
	1	114.833	158.849
	2	0.075	0.075
2	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**
	1	114.833	158.849
	2	0.075	0.075
4	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**
	1	114.833	158.849
	2	0.075	0.075
5	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**
	1	114.833	158.849
	2	0.075	0.075
8	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**
	1	114.833	158.849
	2	0.075	0.075
10	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**
	1	114.833	158.849
	2	0.075	0.075
20	3	2.608	3.399
	4	118.167	8.125
	5	117.342	182.575
Mean		**70.605**	**70.605**
Variance		**3201.187**	**6746.090**

Note: Mean refers to the simulation mean of estimators $\hat{\mu}_{acs}$ and $\bar{y}_{sys}$.

Table 3.7 The Simulation Mean and MSE of $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$, $\hat{v}_{sys(2)}(\bar{y})$ When the Real Blue-Winged Teal Population Consists of $N=10$ PSUs.

p	**Possible PSU chosen**	$\hat{v}_{acs(1)}(\hat{\mu})$	$\hat{v}_{acs(2)}(\hat{\mu})$	$\hat{v}_{sys(1)}(\bar{y})$	$\hat{v}_{sys(2)}(\bar{y})$
2	1	0.056	-	0.056	-
	2	0.020	-	0.020	-
	3	0.000	-	0.000	-
	4	22249.799	-	24.130	-
	5	10966.841	-	114683.326	-
	6	21369.729	-	90388.902	-
	7	0.000	-	0.000	-
	8	12.087	-	33.830	-
	9	11.769	-	36.141	-
	10	24.406	-	0.027	-
Mean		**5463.47**	**-**	**20516.64**	**-**
MSE		**189161255.79**	**-**	**104384950.99**	**-**
4	1	0.056	-	0.056	-
	2	0.020	-	0.020	-
	3	0.000	-	0.000	-
	4	22018.885	-	24.079	-
	5	12548.469	-	114648.258	-
	6	22298.869	-	90395.348	-
	7	0.000	-	0.000	-
	8	11.960	-	33.799	-
	9	11.552	-	31.251	-
	10	24.454	-	0.028	-
Mean		**5691.43**	**-**	**20513.28**	**-**
MSE		**72789251.26**	**-**	**182680129.60**	**-**
5	1	0.056	-	0.056	-
	2	0.0004	-	0.020	-
	3	0.000	-	0.000	-
	4	22904.927	-	24.249	-
	5	12789.197	-	114625.218	-
	6	22529.469	-	90396.829	-
	7	0.000	-	0.000	-
	8	11.726	-	33.110	-
	9	11.756	-	36.110	-
	10	24.455	-	0.028	-
Mean		**5827.159**	**-**	**20515.279**	**-**
MSE		**61085140.86**	**-**	**182346863.60**	**-**
10	1	0.056	0.056	0.056	0.056
	2	0.0004	0.020	0.020	0.020
	3	0.000	0.000	0.000	0.000
	4	22529.043	22904.507	24.058	24.276
	5	13661.817	14915.501	114660.485	114987.214
	6	22561.815	22941.246	90397.814	90395.605
	7	0.000	0.000	0.000	0.000
	8	11.599	11.925	33.524	33.526
	9	11.634	11.909	35.802	35.648
	10	24.459	24.445	0.028	0.028
Mean		**5880.04**	**6080.961**	**20515.18**	**20547.637**
MSE		**39048720.75**	**22962500.02**	**180760072.03**	**203445416.20**

Note: Mean refers to the simulation mean of $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$, $\hat{v}_{sys(2)}(\bar{y})$.

Table 3.8 The Simulation Mean and MSE of $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$, $\hat{v}_{sys(2)}(\bar{y})$ When the Real Blue-Winged Teal Population Consists of $N=5$ PSUs.

p	Possible PSU chosen	$\hat{v}_{acs(1)}(\hat{\mu})$	$\hat{v}_{acs(2)}(\hat{\mu})$	$\hat{v}_{sys(1)}(\bar{y})$	$\hat{v}_{sys(2)}(\bar{y})$
	1	5179.359	-	20089.996	-
	2	0.0045	-	0.0045	-
2	3	2.656	-	7.499	-
	4	4880.370	-	13.093	-
	5	2499.187	-	25516.48	-
Mean		**2512.315**	-	**9125.47**	-
MSE		**27700824.09**	-	**76195014.41**	-
	1	5168.986	-	20088.592	-
	2	0.0045	-	0.0045	-
4	3	2.685	-	7.517	-
	4	5047.413	-	12.789	-
	5	2950.213	-	25517.887	-
Mean		**2633.86**	-	**9125.358**	-
MSE		**13049298.50**	-	**76047866.73**	-
	1	5137.788	-	20088.993	-
	2	0.0045	-	0.0045	-
5	3	2.653	-	7.497	-
	4	5046.251	-	12.543	-
	5	3012.559	-	25526.513	-
Mean		**2639.85**	-	**9127.11**	-
MSE		**16006235.98**	-	**76071427.63**	-
	1	5108.632	-	20088.537	-
	2	0.0045	-	0.0045	-
8	3	2.647	-	7.493	-
	4	5129.201	-	13.158	-
	5	3082.797	-	25510.203	-
Mean		**2664.66**	-	**9123.88**	-
MSE		**14556932.43**	-	**72293301.54**	-
	1	5136.152	-	20089.014	-
	2	0.0045	-	0.0045	-
10	3	2.673	-	7.509	-
	4	5082.953	-	13.005	-
	5	3141.166	-	25495.055	-
Mean		**2672.59**	-	**9120.92**	-
MSE		**14136331.78**	-	**75870180.76**	-
	1	5117.414	5186.067	20088.811	20089.167
	2	0.0045	0.045	0.0045	0.0045
20	3	2.647	2.689	7.491	7.499
	4	5139.842	5139.842	12.994	12.899
	5	3267.939	3405.324	25498.518	25554.029
Mean		**2705.57**	**2746.793**	**9121.563**	**9132.719**
MSE		**13503941.49**	**14658764.36**	**72205928.98**	**124205590.00**

Note: Mean refers to the simulation mean of $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$, $\hat{v}_{sys(2)}(\bar{y})$.

Table 3.9 The Calculation of Relative Bias (RB) of $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ When the Real Blue-Winged Teal Population Consists of $N=10$ and $N=5$ PSUs.

N **&** M	p	$RB\left[\hat{v}_{acs(1)}(\hat{\mu})\right]$	$RB\left[\hat{v}_{acs(2)}(\hat{\mu})\right]$	$RB\left[\hat{v}_{sys(1)}(\bar{y})\right]$	$RB\left[\hat{v}_{sys(2)}(\bar{y})\right]$
10 & 20	2	-0.4895	-	0.1133	-
	4	-0.4776	-	0.1128	-
	5	-0.4651	-	0.1126	-
	10	-0.4603	-0.442	0.1129	0.1146
5 & 40	2	-0.2080	-	0.3525	-
	4	-0.1873	-	0.3525	-
	5	-0.1838	-	0.3527	-
	8	-0.1656	-	0.3522	-
	10	-0.1649	-	0.3518	-
	20	-0.1574	-0.1419	0.3519	0.3536

Table 3.10 The Relative Frequency of Coverage μ of 95 % Confidence Interval That is Produced by $\hat{v}_{acs(1)}(\hat{\mu})$, $\hat{v}_{acs(2)}(\hat{\mu})$, $\hat{v}_{sys(1)}(\bar{y})$ and $\hat{v}_{sys(2)}(\bar{y})$ When the Real Blue-Winged Teal Population Consists of $N=10$ and $N=5$ PSUs.

N **&** M	p	$CI\left[\hat{v}_{acs(1)}(\hat{\mu})\right]$	$CI\left[\hat{v}_{acs(2)}(\hat{\mu})\right]$	$CI\left[\hat{v}_{sys(1)}(\bar{y})\right]$	$CI\left[\hat{v}_{sys(2)}(\bar{y})\right]$
10 & 20	2	0.118	-	0.200	-
	4	0.257	-	0.200	-
	5	0.300	-	0.300	-
	10	0.300	0.300	0.300	0.300
5 & 40	2	0.245	-	0.400	-
	4	0.600	-	0.400	-
	5	0.600	-	0.400	-
	8	0.600	-	0.400	-
	10	0.600	-	0.400	-
	20	0.600	0.600	0.400	0.400

CHAPTER 4

PARTIALLY SYSTEMATIC ADAPTIVE CLUSTER SAMPLING

4.1 Introduction

Recall that the adaptive cluster sampling design with primary and secondary sampling units was proposed by Thompson (1991: 1103-1105). It is recommended to be applied with sample sizes of at least two primary sampling units (PSUs). However, a single primary sampling unit is needed for some field work but limitations of estimation occurred when a single primary sampling unit was chosen; that is, an unbiased estimator of variance of the estimated of the population mean is not available though we can obtain an unbiased estimator of the population mean. So one of the points of interest is how we can obtain an unbiased estimator of the population mean and its variance. In conventional systematic sampling, one method often used to handle with variance estimation is a method of composition. In this method, the sample size is split into two parts and the systematic sampling is implemented to select the first sample, while another sample is drawn from remaining units in population by adopting any sampling design (Hedayat and Sinha, 1991: 239-241). This method is often called the partially systematic sampling. Partially systematic sampling procedure has been proposed by Zinger in 1984. The procedure is to start up by the selection of a systematic sample or a single primary sampling unit at random, then to select a supplement sample that is a set of units selected from the remaining units in the population, either a simple random sampling or systematic sampling. By this procedure, an unbiased estimator of the variance of the sample mean can be

.

obtained. But the restriction of the procedure proposed by Zinger is that the sample size is required to be a multiple of the sample size used (Leu and Tsui, 1996: 617-630). The new sampling procedure was modified and was proposed by Leu and Tsui (1996: 617-630) in order to reduce the restriction. This procedure is called the new partially systematic sampling. In this study, the basic concept of the partially systematic sampling proposed by Zinger was applied to adaptive cluster sampling design. However, the sampling scheme of selection of a single primary sampling unit and units in the supplementary sample can be done with varying probability. Section 4.2 describes the design of partially systematic adaptive cluster sampling when a selection is without replacement of units, without replacement of networks, and without replacement of clusters. In these designs, the population consists of MN secondary units, and (i, j) denotes the j^{th} secondary sampling unit of the i^{th} primary sampling unit in the population; $j = 1,2,...,M$ and $i = 1,2,...,N$. Let $k = 1,2,...,K$ be the network label of the population, and let i represent the i^{th} selection of units in the sample, where $i = 1,2,...,m$.

The parameter of interest is the population mean, $\mu = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} y_{ij}}{NM} = \frac{\tau}{NM}$, and the proposed estimator of μ and its variance estimators are presented in section 4.3. The simulation study has been carried out with real blue-winged teal data which was given by Smith et al. (1995: 777-778). In simulation study, the primary sampling unit was defined as a set of secondary sampling units being arranged in strip pattern. The efficiency comparison of proposed estimators was investigated based on simulation study. The simulation study result is represented in section 4.4

4.2 Design and Terminology

In the design of partially systematic adaptive cluster sampling (partially SACS), the sampling scheme of selection without replacement of units (Raj, 1956:

269-284), without replacement of networks (Salehi and Seber, 1997: 209-219) and without replacement of clusters (Dryver and Thompson, 2007: 35-43), is applied to select an initial sample which consists of a single primary sampling unit and a number of secondary sampling units. Details of these sampling schemes of the selection are as follows:

4.2.1 Sampling Without Replacement of Units

In the design of partially SACS, an initial sample of size m is composed of a single primary sampling unit and $m_0 = m-1$ secondary sampling units, where $m \geq 2$. The first initial unit, which is a single primary sampling unit, is drawn from the population at random. After a single random start is selected, all M secondary sampling units corresponding to the same position with the random start are added automatically to the sample and if their y-values satisfy the condition of interest then their associated networks and edge units will be added to the final sample. If any one of these secondary sampling units do not satisfy the condition, then it is considered as a network of size one. For the second initial unit, a secondary unit will be selected from the population exclusive of all of M secondary sampling units within the previous initial unit. Its associated network and edge units will be added to the final sample if they satisfy the condition. None of the selected secondary sampling units in the previous selection can be selected repeatedly. In turn, each m of the initial unit will be selected at random from the population exclusive of all of secondary sampling units in the previous selection.

4.2.2 Sampling Without Replacement of Networks

In this design, the first initial unit is a single primary sampling unit that is drawn from the population at random. If any network includes at least one of the secondary sampling units within the selected primary sampling unit, then that network and its associated edge units will be added to the final sample. All these networks will be deleted from the population before the next draw. For the second initial unit, a secondary sampling unit will be selected from the population exclusive of all secondary sampling units within all networks included in the previous selection. Note that none of any network selected in the previous draw can be repeated selection. For any edge unit, it can be repeated selection except the edge unit which was the first initial unit. So for each initial *m* unit, it will be selected by random from the population exclusive of all networks in the previous selection.

4.2.3 Sampling Without Replacement of Clusters

In this design, the first initial unit will be selected with the same procedure as the sampling without replacement of networks. Whenever any secondary sampling unit within it satisfies the condition, its associated cluster, which is composed of a network and its associated edge units, will be added to the sample. These clusters will be deleted from the population before the second selection will be made. For the second initial unit, a secondary sampling unit will be selected from the population exclusive of all distinct secondary sampling units obtained at the first draw. This means that the selected networks and their associated edge units can not be repeatedly selected.

Figure 4.1A shows the first initially sampled unit consisting of a set of 9 secondary sampling units being arranged in systematic pattern. Adaptively added units will be occurred if they satisfy the condition and then their associated cluster will be added to the sample. This associated cluster shows edge units in light gray (8 light gray squares) and the network of units in dark gray squares (4 dark gray squares). A unit in double lines which is labeled with the gray color is a network of size one. In figure 4.1B, presenting sampling without replacement of units, all 9 light gray squares will be deleted from the population before the 2nd draw. The second initial unit which is a unit in double lines is selected by random from the population. Its cluster which is a set of units in dash line is created and is added to the sample. However, only this initially sampled unit will be deleted before the next draw. Sampling without replacement of networks is presented in figure 4.1C. It shows that the second initial unit will be selected from the population exclusive of all 12 light gray squares. The second initially sampled unit and its cluster which is a set of units in dash line is created and is added to the sample. But only its network (3 dark squares including its self) will be deleted before the next draw. Figure 4.1D shows sampling without replacement of clusters, where the second initial unit will be selected exclusive of all 19 light gray squares (a cluster of size 12 and 7 clusters of size one). This sampled unit is a double line with gray labels. A set of 4 units with a dash plus its self forms a cluster. This cluster will be added to the sample and will be deleted from the population before the next draw.

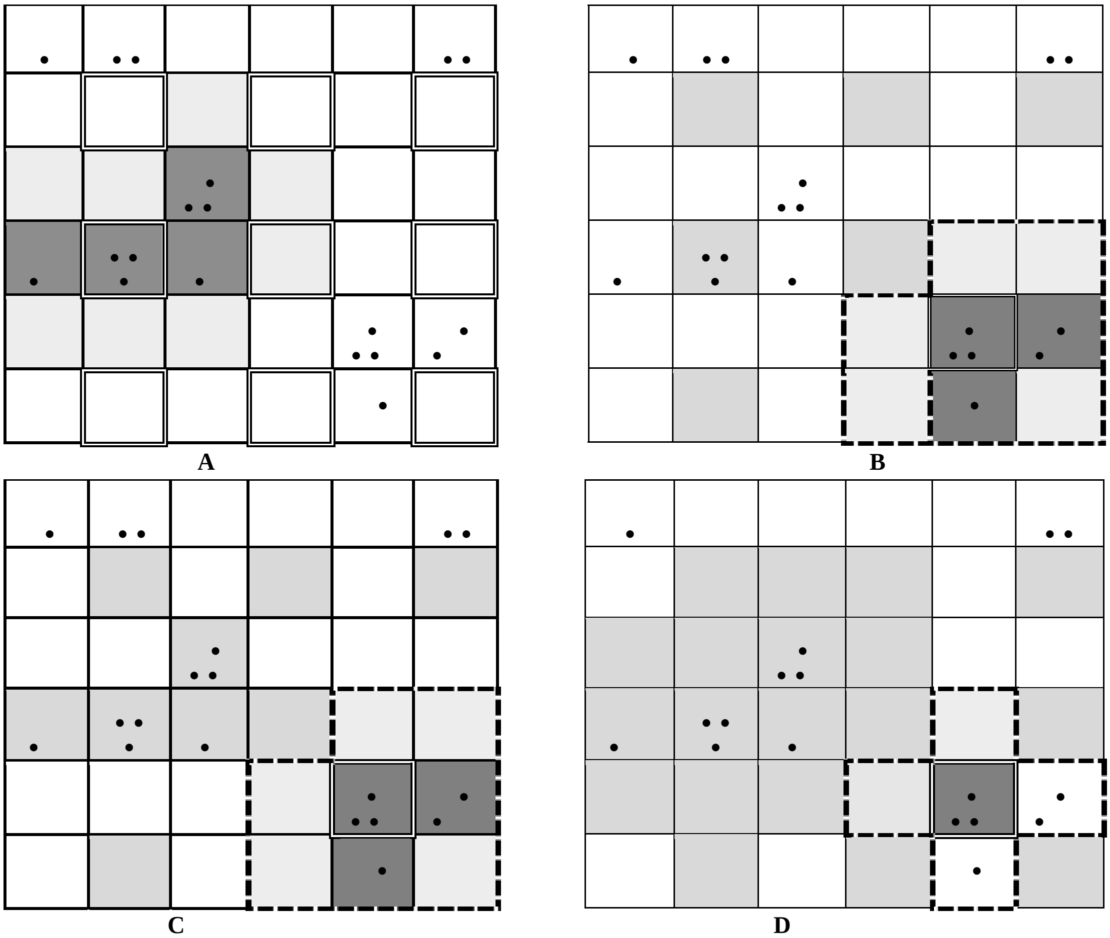


Figure 4.1 (A) Shows the Selected Primary Sampling Unit (the First Initially Sampled Unit), Networks and Their Associated Edge Units, and Their Associated Cluster. (B) Shows the Results When Sampling Without Replacement of Units was Done. (C) Shows the Results When Sampling Without Replacement of Networks was Done. (D) Shows the Results When Sampling Without Replacement of Clusters was Done.

4.3 New Estimators

4.3.1 A Modified Raj Type Estimator

The Raj type estimator, usually used with the design where the probability for the unit drawn in each draw is different, is

$$\hat{\mu}_D = \frac{1}{Nm}\sum_{i=1}^{m} t_i \quad , \tag{4.1}$$

where $t_1 = \dfrac{y_1}{p_1}$ and $t_i = \sum_{j=1}^{i-1} y_j + \dfrac{\left(1-\sum_{j=1}^{i-1} p_j\right) y_i}{p_i}$, for $i = 2, 3, \ldots, m$. The probability p_i represents the first-draw probability for unit i and $p_i \Big/ \left(1-\sum_{i=1}^{m-1} p_i\right)$ is the conditional probability of selecting the i^{th} unit in the sample given the first $i-1$ prior selections.

4.3.2 When the Design is a Selection Without Replacement of Units

When an initial unit is selected without replacement of units, it is necessary to modify the Raj type estimator. Let the first-draw probability for each primary sampling unit chosen be $p_1 = 1/N$. The selected primary sampling unit obtained from the selection is called the first initial unit.

For $i = 1$ or the 1^{st} selection, let S_1 be a set of secondary sampling units label within the 1^{st} initial unit, $y_{j.}$ be total y-values of network corresponding to secondary sampling unit j within the 1^{st} initial unit, $\bar{y}_j$ be an average of network corresponding to secondary sampling unit j , and m_j be the total number of secondary sampling units within the network including the j secondary sampling unit within the 1^{st} initial unit, and let

$$Z_i = \frac{\sum_{j \in S_1} y_{j.} \big/ m_j}{p_1} = \frac{\sum_{j \in S_1} \bar{y}_j}{p_1} \tag{4.2}$$

For $i = 2,3,...,m$, let $p_i' = 1/MN$ and the conditional probability for getting a secondary sampling unit in the i^{th} draw given the first $i-1$ prior draw be $p_i' \Big/ \left(1 - \sum_{j \in S_{i-1}} p_j'\right)$, and let

$$Z_i = \sum_{j \in S_{i-1}} \bar{y}_j + \frac{\left(1 - \sum_{j \in S_{i-1}} p_j'\right) \bar{y}_i}{p_i'} \quad , \tag{4.3}$$

where S_{i-1} is a set of secondary sampling units label obtained from the first $i-1$ prior selection, and $\bar{y}_i$ is the average of network including the i^{th} unit of the sample. When $i-1=1$, $\sum_{j \in S_1} p_j' = p_1$ and when $i-1 \geq 2$, where $i = 3,4,...,m$, notice that there is only one network including the i^{th} unit of the initial sample so that $\sum_{j \in S_i} \bar{y}_j = \bar{y}_i$ and $\sum_{j \in S_i} p_j' = p_i'$.

Theorem 4.1 For partially systematic adaptive cluster sampling, if the initial sample of a single primary sampling unit and $m-1$ secondary sampling units is selected without replacement of units, then an estimator (based on Raj, 1956: 269-284)

$$\hat{\mu}_{units} = \frac{1}{MN} \frac{1}{m} \sum_{i=1}^{m} Z_i \tag{4.4}$$

is an unbiased estimator of the population mean, where

$$
\begin{aligned}
Z_1 &= \frac{\sum_{j\in S_1} y_{j.}/m_j}{p_1} = \frac{\sum_{j\in S_1} \bar{y}_j}{p_1} \\
Z_2 &= \sum_{j\in S_1} \bar{y}_j + \frac{\left(1-\sum_{j\in S_1} p'_j\right)\bar{y}_2}{p'_2} \\
&\vdots \\
Z_m &= \left(\sum_{j\in S_1} \bar{y}_j + \bar{y}_2 + \ldots + \bar{y}_{m-1}\right) + \frac{\left(1-\sum_{j\in S_1} p'_j - p'_2 - \ldots - p'_{m-1}\right)\bar{y}_m}{p'_m}
\end{aligned} \tag{4.5}
$$

with the variance of $\hat{\mu}_{units}$ is given by

$$V(\hat{\mu}_{units}) = \frac{1}{(MN)^2}\frac{1}{m^2}\sum_{i=1}^{m} V(Z_i) \tag{4.6}$$

Further, an unbiased estimator of $V\left(\hat{\mu}_{units}\right)$ is given by

$$\hat{v}(\hat{\mu}_{units}) = \frac{1}{m(m-1)}\sum_{i=1}^{m}\left(\frac{Z_i}{NM} - \hat{\mu}_{units}\right)^2 \tag{4.7}$$

Proof

First, to claim that $\hat{\mu}_{units}$ in (4.4) is an unbiased estimator of the population mean.

To establish this claim, let $U = \left\{(i', j') : 1 \le i' \le N,\ 1 \le j' \le M\right\}$, where j' represents secondary sampling unit labels and i' represents primary sampling unit labels in the population. Let $Y_{i'.}$ be the total value of secondary sampling units in the i'^{th} primary sampling unit and $\tau = \sum_{i'=1}^{N} Y_{i'.}$ be the population total.

Suppose an ordered sample $s_0 = \left(i_1, i_2, \ldots, i_m\right)$ is draw utilizing the sampling without replacement of units, where $m \ge 2$. Notice that unit obtained in the 1^{st}

selection, $i=1$, denote with i_1. Let the unit obtained in the i^{th} selection be denoted by i_i. Let j denote secondary sampling unit labels in the initial unit obtained from the i^{th} selection, $y_{j\cdot}$ be the total value of network corresponding unit j, and m_j be the total secondary sampling units in the network associated with $y_{j\cdot}$.

When $i=1$, a single primary sampling unit of size M secondary sampling units is drawn from the population and $1 \le i_1 \le N$. The selected primary sampling unit is called the first initial unit. Let S_{i_1} denote a set of secondary sampling units within the first initial unit, $y_{j\cdot}$ be defined as the total value of the network including unit j in S_{i_1}, and $y_{i_1\cdot}$ be the total value of all the networks associated with all units in S_{i_1}. For $i \ge 2$ and $i \le m$, let $S_{i_{i-1}}$ be a set of secondary sampling units obtained from the first $(i-1)$ prior selection.

The expectation of $\hat{\mu}_{units}$ is

$$E\left(\hat{\mu}_{units}\right) = E\left(\sum_{i=1}^{m} Z_i \Big/ NMm\right) = \sum_{i=1}^{m} E\left(Z_i\right) \Big/ NMm \tag{4.8}$$

Notice that each estimator Z_i, which is defined in equation (4.5), is a function of the sample value drawn.

So based on Raj (1956: 271), the expected value Z for general initial unit m or $E\left(Z_m\right)$ is

$$E\left(Z_m\right) = E_1 E_2 \ldots E_m\left(Z_m\right) \tag{4.9}$$

This needs work with a special case of $m=2$, a single primary sampling unit and a secondary sampling unit, is selected without replacement of units. The estimator of the population mean is of the form

$$\hat{\mu}_{units} = \frac{1}{2MN}\left(Z_1 + Z_2\right) \tag{4.10}$$

is suggested to be an unbiased estimator of μ since

$$E\left(\hat{\mu}_{units}\right)=\frac{1}{2NM}\left\{E\left(Z_1\right)+E\left(Z_2\right)\right\}=\frac{\tau}{NM}=\mu \qquad (4.11)$$

and both estimators Z_1 and Z_2 are unbiased estimator of τ .

Considering the terms of the expected value of Z_1 and Z_2. Based on the first initial unit, i_1, the estimator Z_1 is of the form

$$Z_1=\sum_{j\in S_{i_1}}\bar{y}_j \Big/ p_{i_1} \qquad (4.12)$$

The expectation of Z_1 is

$$E_1\left(Z_1\right)=E_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j \Big/ p_{i_1}\right\} \qquad (4.13)$$

where E_1 is the expected value for all possible primary sampling units obtained in the first selection.

$$=\sum_{i_1=1}^{N}\left(\frac{\sum_{j\in S_{i_1}} y_{j\cdot}/m_j}{p_{i_1}}\right)\Pr\left(i_1\right)$$

$$=\sum_{i_1=1}^{N}\left(\frac{\sum_{j\in S_{i_1}} y_{j\cdot}\big/ m_j}{\left(1/N\right)}\right)\left(\frac{1}{N}\right)=\sum_{i_1=1}^{N}\left(\sum_{j\in S_{i_1}} y_{j\cdot}/m_j\right)$$

Notice that if $m_j=1$ for all $j\in S_{i_1}$ and for all S_{i_1} obtained from the 1st selection, then $\sum_{j\in S_{i_1}} y_{j\cdot}\big/ m_j=\sum_{j\in S_{i_1}} y_{j\cdot}=Y_{i_1\cdot}$. If $m_j>1$ for some $j\in S_{i_1}$ and the network associated with j consists of units belong to some S_{i_1}, then $\sum_{i_1=1}^{N}\left(\sum_{j\in S_{i_1}} y_{j.}/m_j\right)=\sum_{i_1=1}^{N} Y_{i_1}$,

This gives the expected value of Z_1 as

$$E_1\left(Z_1\right)=\sum_{i_1=1}^{N} Y_{i_1.} = \tau = \text{the population total} \tag{4.14}$$

Next, based on the first two units i_1 and i_2, the estimator Z_2 is of the form

$$Z_2 = \sum_{j\in S_{i_1}} \bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\bar{y}_{i_2}}{p'_{i_2}} \tag{4.15}$$

This estimator is also unbiased for τ since the conditional expectation of Z_2 given i_1 (based on Raj, 1956: 271) is

$$E\left(Z_2\right)=E_1E_2\left(\sum_{j\in S_{i_1}} \bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right), \tag{4.16}$$

where E_1 is the expected values for all possible primary sampling units obtained in the first selection, E_2 is the expected values for all remaining secondary sampling units in the population excluding all secondary sampling units in S_{i_1}.

$$= E_1\left\{E_2\left(\sum_{j\in S_{i_1}} \bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\middle| i_1\right)\right\}$$

$$= E_1\left\{\sum_{k\in S_{i_1}}\bar{y}_j + E_2\left(\left.\frac{\left(1-\sum_{k\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right| i_1\right)\right\}$$

$$= E_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j + \sum_{i_2\in U-S_{i_1}}\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\Pr\left(i_2\middle|i_1\right)\right\}$$

$$= E_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j + \left(\sum_{i_2\in U-S_{i_1}}\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\times\frac{p'_{i_2}}{\left(1-\sum_{j\in S_{i_1}}p'_j\right)}\right)\right\}$$

$$= E_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j + \sum_{i_2\in U-S_{i_1}}\bar{y}_{i_2}\right\} = E_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j + \left(\tau - \sum_{j\in S_{i_1}}\bar{y}_j\right)\right\} = \tau$$

This gives

$$E\left(Z_2\right) = E_1E_2\left(\sum_{j\in S_{i_1}}\bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right) = \tau \qquad (4.17)$$

So that based on the initial units $\left(i_1, i_2, ..., i_m\right)$, the conditional expectation of Z_m given $\left(i_1, i_2, ..., i_{m-1}\right)$ is $E\left(Z_m\right) = E_1E_2...E_m\left(Z_m\middle|i_1, i_2, ...i_{m-1}\right) = \tau$.

This gives that the expectation of $\hat{\mu}_{units}$ is

$$E\left(\hat{\mu}_{units}\right)=\frac{1}{mNM}\sum_{i=1}^{m}E\left(Z_i\right)$$

$$=\frac{1}{mNM}\sum_{i=1}^{m}\tau=\frac{m\tau}{mNM}=\frac{\tau}{MN}=\mu \qquad (4.18)$$

Clearly, $\hat{\mu}_{units}$ is an unbiased estimator of the population mean.

For the true variance of estimator $\hat{\mu}_{units}$,

$$V\left(\hat{\mu}_{units}\right)=V\left(\frac{1}{mNM}\left(\sum_{i=1}^{m}Z_i\right)\right)=\frac{1}{m^2\left(NM\right)^2}V\left(\sum_{i=1}^{m}Z_i\right) \qquad (4.19)$$

$$=\frac{1}{m^2\left(NM\right)^2}\left[\sum_{i=1}^{m}V\left(Z_i\right)+\sum_{i=1}^{m}\sum_{j\neq i}^{m}Cov\left(Z_i,Z_j\right)\right]$$

Under this scheme of the selection, the unit selection in each draw is taken independently, and based on Raj (1956: 271-273) and Hedayat and Sinha (1991: 135-136), the covariance between any pair of the estimators $Z_1,Z_2,...,Z_m$ is zero, $Cov\left(Z_i,Z_j\right)=0$ for all $i\neq j$. Thus the true variance of estimator $\hat{\mu}_{units}$ will become

$$V\left(\hat{\mu}_{units}\right)=\frac{1}{m^2\left(NM\right)^2}\sum_{i=1}^{m}V\left(Z_i\right) \qquad (4.20)$$

Because estimator Z_i is a function of the sample values drawn. Based on theorem 2 (Raj, 1956: 271), we get the variance of Z_m as

$$V\left(Z_m\right)=\left(E_1V_{23...m}+V_1E_{23...m}\right)\left(Z_m\right) \qquad (4.21)$$

Considering to a special case of $m=2$ and using (4.20), the true variance of estimator $\hat{\mu}_{units}$ is

$$V\left(\hat{\mu}_{units}\right)=\frac{1}{m^2\left(NM\right)^2}\left\{V\left(Z_1\right)+V\left(Z_2\right)\right\} \tag{4.21}$$

For $i=1=i_1$, the variance of Z_1 is of the form

$$V\left(Z_1\right)=V\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)=V\left(\frac{\sum_{j\in S_{i_1}}y_{j.}/m_j}{p_{i_1}}\right) \tag{4.22}$$

$$=E_1\left\{\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}-E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)\right\}^2$$

$$=E_1\left\{\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)^2-2E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)+\left[E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)\right]^2\right.$$

Following in (4.14), we get

$$=E_1\left\{\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)^2-2\tau\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)+\tau^2\right\}$$

$$=E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)^2-2\tau E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)+E_1\left(\tau^2\right)$$

$$=E_1\left(\frac{\sum_{j\in S_{i_1}}\overline{y}_j}{p_{i_1}}\right)^2-2\tau^2+\tau^2$$

$$= \left\{ \sum_{i_1=1}^{N} \left(\frac{\sum_{j \in S_{i_1}} \bar{y}_j}{p_{i_1}} \right)^2 \Pr(i_1) \right\} - 2\tau^2 + \tau^2$$

$$= \left\{ \sum_{i_1=1}^{N} \left(\frac{\sum_{j \in S_{i_1}} \bar{y}_j}{p_{i_1}} \right)^2 p_{i_1} \right\} - \tau^2$$

This gives the true variance of Z_1 as

$$V(Z_1) = \left\{ \sum_{i_1=1}^{N} \frac{\left(\sum_{j \in S_{i_1}} \bar{y}_j \right)^2}{p_{i_1}} \right\} - \tau^2 \tag{4.23}$$

For $i = 2 = i_2$, the equation (4.21) is used to find the variance of Z_2. This gives the variance of Z_2 as equal to the sum of the expected value of the conditional variances and the variance of the conditional expected values as follows:

$$V(Z_2) = (E_1V_2 + V_1E_2) \left(\sum_{j \in S_{i_1}} \bar{y}_j + \frac{\left(1 - \sum_{j \in S_{i_1}} p'_j \right) \bar{y}_{i_2}}{p'_{i_2}} \right) \tag{4.24}$$

where E_1 is the expected value for all possible primary sampling units obtained in the first draw, E_2 is the expected value for all remaining secondary sampling units in the population excluding all units in S_{i_1}, V_1 is the variance for all possible primary sampling units obtained in the first draw, and V_2 is the conditional variance for Z_2.

$$= E_1V_2\left(\sum_{j\in S_{i_1}}\bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right) + V_1E_2\left(\sum_{j\in S_{i_1}}\bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right)$$

Consider in terms of $E_1V_2\left(\sum_{j\in S_{i_1}}\bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right)$

$$= E_1\left\{V_2\left(\sum_{j\in S_{i_1}}\bar{y}_j + \frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right)\right\} = E_1\left\{V_2\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right)\right\} \qquad (4.25)$$

$\because\ V_2\left(\sum_{j\in S_{i_1}}\bar{y}_j\right) = 0$, and

$$= E_1\left\{E_2\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}} - E_2\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right)\right)^2\right\}$$

Since $E_2\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\right) = \left(\tau - \sum_{j\in S_{i_1}}\bar{y}_j\right)$ so we get

$$= E_1\left\{ \begin{array}{l} E_2\left(\left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2}}{p'_{i_2}}\right)^2\right) - 2\left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right) E_2\left(\left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2}}{p'_{i_2}}\right)\right) \\ + \left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2 \end{array} \right\}$$

$$= E_1\left\{ E_2\left(\left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2.}}{p'_{i_2}}\right)^2\right) - 2\left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2 + \left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2 \right\}$$

$$= E_1\left\{ \left[\sum_{i_2\in U-S_{i_1}} \left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2}}{p'_{i_2}}\right)^2 \Pr\left(i_2 \,|\, i_1\right)\right] - \left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2 \right\}$$

$$= E_1\left\{ \left(\sum_{i_2\in U-S_{i_1}} \left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2}^2}{p'_{i_2}}\right)\right) - \left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2 \right\}$$

$$= \sum_{i_1=1}^{N} \Pr\left(i_1\right) \sum_{i_2\in U-S_{i_1}} \left(\frac{\left(1-\sum_{j\in S_{i_1}} p'_j\right)\overline{y}_{i_2}^2}{p'_{i_2}}\right) - \sum_{i_1=1}^{N} \Pr\left(i_1\right)\left(\tau - \sum_{j\in S_{i_1}} \overline{y}_j\right)^2$$

$$= \sum_{i_1=1}^{N} p_{i_1} \sum_{i_2 \in U - S_{i_1}} \left(\frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}^2}{p'_{i_2}} \right) - \sum_{i_1=1}^{N} p_{i_1} \left(\tau - \sum_{j \in S_{i_1}} \overline{y}_j \right)^2$$

This gives

$$E_1 V_2 \left(Z_2 \right) = \sum_{i_1=1}^{N} p_{i_1} \sum_{i_2 \in U - S_i} \left(\frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}^2}{p'_{i_2}} \right) - \sum_{i_1=1}^{N} p_{i_1} \left(\tau - \sum_{j \in S_{i_1}} \overline{y}_j \right)^2 \qquad (4.26)$$

Next, consider in terms of $V_1 E_2 \left(Z_2 \right) = V_1 E_2 \left(\sum_{j \in S_{i_1}} \overline{y}_j + \frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}}{p'_{i_2}} \right)$

$$= V_1 \left\{ E_2 \left(\sum_{j \in S_{i_1}} \overline{y}_j + \frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}}{p'_{i_2}} \right) \right\} \qquad (4.27)$$

$$= V_1 \left\{ \sum_{j \in S_{i_1}} \overline{y}_j + E_2 \left(\frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}}{p'_{i_2}} \right) \right\}$$

$$= V_1 \left\{ \sum_{j \in S_{i_1}} \overline{y}_j + \sum_{i_2 \in U - S_{i_1}} \frac{\left(1 - \sum_{j \in S_{i_1}} p'_j\right) \overline{y}_{i_2}}{p'_{i_2}} \Pr\left(i_2 \middle| i_1 \right) \right\}$$

$$=V_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j+\sum_{i_2\in U-S_{i_1}}\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}}{p'_{i_2}}\left(\frac{p'_{i_2}}{\left(1-\sum_{j\in S_{i_1}}p'_j\right)}\right)\right\}$$

$$=V_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j+\sum_{i_2\in U-S_{i_1}}\bar{y}_{i_2}\right\}=V_1\left\{\sum_{j\in S_{i_1}}\bar{y}_j+\left(\tau-\sum_{j\in S_{i_1}}\bar{y}_j\right)\right\}=V_1\{\tau\}=0$$

so that we get the true variance of Z_2 as

$$V(Z_2)=\sum_{i_1=1}^{N}p_{i_1}\sum_{i_2\in U-S_{i_1}}\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j\right)\bar{y}_{i_2}^2}{p'_{i_2}}\right)-\sum_{i_1=1}^{N}p_{i_1}\left(\tau-\sum_{j\in S_{i_1}}\bar{y}_j\right)^2 \tag{4.28}$$

where i_1 and i_2 are units obtained in the 1^{st} and the 2^{nd} draws.

Thus, the true variance for initial unit m, Z_m, is

$$\begin{aligned}V(Z_i)=\sum_{i_1=1}^{N}p_{i_1}\sum_{i_2\in U-S_{i_1}}p'_{i_2}\cdots\sum_{i_m\in U-S_{i_1}-\ldots-S_{i_{m-1}}}\left(\frac{\left(1-\sum_{j\in S_{i_1}}p'_j-p'_{i_2}\ldots-p'_{i_{m-1}}\right)\bar{y}_{i_m}^2}{p'_{i_m}}\right)\\-\sum_{i=1}^{N}p_i\sum_{i_2\in U-S_{i_1}}p'_{i_2}\cdots\sum_{i_m\in U-S_{i_1}-\ldots-S_{i_{m-1}}}\left(\tau-\sum_{j\in S_{i_1}}\bar{y}_j-\bar{y}_{i_2}\ldots-\bar{y}_{i_{m-1}}\right)^2\end{aligned} \tag{4.29}$$

To confirm that $\hat{v}(\hat{\mu}_{units})$ in (4.7) is an unbiased estimator of $V(\hat{\mu}_{units})$ in (4.6). Starting with the expectation of $\hat{v}(\hat{\mu}_{units})$,

$$E\left(v\left(\hat{\mu}_{units}\right)\right)=E\left\{\frac{1}{m\left(m-1\right)}\sum_{i=1}^{m}\left(\frac{Z_i}{NM}-\hat{\mu}_{units}\right)^2\right\} \tag{4.30}$$

$$=E\left\{\frac{1}{m\left(m-1\right)}\sum_{i=1}^{m}\left[\left(\frac{Z_i}{NM}\right)^2-2\left(\frac{Z_i}{NM}\right)\hat{\mu}_{units}+\hat{\mu}_{units}^2\right]\right\}$$

$$=E\left\{\frac{1}{m\left(m-1\right)}\left[\sum_{i=1}^{m}\left(\frac{Z_i}{NM}\right)^2-2\sum_{i=1}^{m}\left(\frac{Z_i}{NM}\right)\hat{\mu}_{units}+\sum_{i=1}^{m}\hat{\mu}_{units}^2\right]\right\}$$

$$=E\left\{\frac{1}{m\left(m-1\right)}\left\{\sum_{i=1}^{m}\frac{Z_i^2}{\left(MN\right)^2}-2m\hat{\mu}_{units}^2+m\hat{\mu}_{units}^2\right\}\right\}$$

$$=E\left\{\frac{\sum_{i=1}^{m}Z_i^2}{\left(MN\right)^2 m\left(m-1\right)}-\frac{m\hat{\mu}_{units}^2}{m\left(m-1\right)}\right\}$$

Since $\frac{m\hat{\mu}_{units}}{m\left(m-1\right)}=\frac{1}{\left(m-1\right)}\left(\frac{\sum_{i=1}^{m}Z_i}{mMN}\right)^2=\frac{1}{m-1}\left(\frac{\sum_{i=1}^{m}Z_i^2+2\sum_{i=1}^{m}\sum_{i<j}^{m}Z_iZ_j}{\left(NM\right)^2 m^2}\right)$ we get

$$=E\left\{\frac{\sum_{i=1}^{m}Z_i^2}{\left(MN\right)^2 m\left(m-1\right)}-\frac{m\hat{\mu}_{units}^2}{m\left(m-1\right)}\right\}$$

$$=\left\{\frac{\sum_{i=1}^{m}E\left(Z_i^2\right)}{\left(MN\right)^2 m\left(m-1\right)}\right\}-\left\{\frac{E\left(\sum_{i=1}^{m}Z_i^2+2\sum_{i=1}^{m}\sum_{i<j}^{m}Z_iZ_j\right)}{\left(MN\right)^2 m^2\left(m-1\right)}\right\}$$

$$=\left\{\frac{\sum_{i=1}^{m}E\left(Z_i^2\right)}{(MN)^2 m(m-1)}\right\}-\left\{\frac{\left(\sum_{i=1}^{m}E\left(Z_i^2\right)+2\sum_{i=1}^{m}\sum_{i<j}^{m}E\left(Z_iZ_j\right)\right)}{(MN)^2 m^2(m-1)}\right\}$$

$$=\left\{\frac{\sum_{i=1}^{m}\left(V(Z_i)+\left(E(Z_i)\right)^2\right)}{(MN)^2 m(m-1)}\right\}-\left\{\frac{\sum_{i=1}^{m}\left(V(Z_i)+\left(E(Z_i)\right)^2\right)}{(MN)^2 m^2(m-1)}\right\}$$

$$-\left\{\frac{2\sum_{i=1}^{m}\sum_{i<j}^{m}\left(Cov\left(Z_i,Z_j\right)-E(Z_i)E\left(Z_j\right)\right)}{(MN)^2 m^2(m-1)}\right\}$$

Using (4.14) and (4.17) we have $E\left(Z_i\right)=\tau$, $E\left(Z_i\right)E\left(Z_j\right)=\tau^2$ and $Cov\left(Z_i,Z_j\right)=0$ so that the expectation of $\hat{v}\left(\hat{\mu}_{units}\right)$ is

$$=\left\{\frac{\sum_{i=1}^{m}\left(V(Z_i)+\tau^2\right)}{(MN)^2 m(m-1)}\right\}-\left\{\frac{\left(\sum_{i=1}^{m}\left(V(Z_i)+\tau^2\right)-2\sum_{i=1}^{m}\sum_{i<j}^{m}\tau^2\right)}{(MN)^2 m^2(m-1)}\right\}$$

$$=\left\{\frac{\sum_{i=1}^{m}V(Z_i)}{(MN)^2 m(m-1)}-\frac{\sum_{i=1}^{m}V(Z_i)}{(MN)^2 m^2(m-1)}\right\}$$

$$+\left\{\frac{m\tau^2}{(MN)^2 m(m-1)}-\frac{m\tau^2}{(MN)^2 m^2(m-1)}-\frac{2\left(m(m-1)/2\right)\tau^2}{(MN)^2 m^2(m-1)}\right\}$$

$$=\left\{\frac{(m-1)\sum_{i=1}^{m}V(Z_i)}{(MN)^2 m^2(m-1)}\right\}+\left\{\frac{m^2\tau^2-m\tau^2-\left(m(m-1)\tau^2\right)}{(MN)^2 m^2(m-1)}\right\}$$

$$\because \left\{ \frac{m^2\tau^2 - m\tau^2 - \left(m\left(m-1\right)\tau^2\right)}{\left(MN\right)^2 m^2 \left(m-1\right)} \right\} = 0 \text{, thus}$$

$$E\left(\hat{v}\left(\hat{\mu}_{units}\right)\right) = \left\{ \frac{1}{\left(MN\right)^2 m^2} \sum_{i=1}^{m} V\left(Z_i\right) \right\} = V\left(\hat{\mu}_{units}\right) \tag{4.31}$$

This means that the estimator $\hat{v}\left(\hat{\mu}_{units}\right)$ is an unbiased estimator of $V\left(\hat{\mu}_{units}\right)$. Next, few formulas without replacement of networks and clusters have similar issues.

4.3.3 When the Design is a Selection Without Replacement of Networks

The first-draw probability $p_1 = 1/N$ for each primary sampling unit is not changed. For $i = 1$, define Z_i^* the same as Z_i in (4.2) as follows:

$$Z_i^* = Z_i = \frac{\sum_{j \in S_1} y_{j.} / m_j}{p_1} = \frac{\sum_{j \in S_1} \bar{y}_j}{p_1} \tag{4.32}$$

For $i = 2,3,...,m$, a secondary sampling unit is selected in each draw from the remaining units in the population exclusive all secondary sampling units within any network obtained in the previous draw. Let $p_i^* = m_{i.}/MN$, where $m_{i.}$ is the total number of secondary sampling units in networks including the i^{th} unit of the sample and MN is the total secondary sampling units in the population. Let conditional probability for each secondary sampling unit in the i^{th} draw given the first-(i-1) prior draw be $p_i^* \big/ 1 - \sum_{k \in S_{i-1}^*} p_k^*$, $y_{i.}^*$ be total y-values of network including the i^{th} unit of the sample and let

$$Z_i^* = \sum_{k \in S_{i-1}^*} y_k^* + \frac{\left(1 - \sum_{k \in S_{i-1}^*} p_k^*\right) y_{i.}^*}{p_i^*} \quad , \tag{4.33}$$

where S_{i-1}^* is a set of all distinct networks included in the first-(i-1) prior selection. When $i-1=1$, $S_{i-1}^* = S_1^*$ represents a set of all distinct networks included in the 1st draw. When $i-1 \geq 2$, notice that the number of networks obtained in draw i is only one network, so $\sum_{k \in S_{i-1}^*} p_k^* = p_{i-1}^*$ and $\sum_{k \in S_{i-1}^*} y_k^* = y_{i-1.}^*$.

Theorem 4.2 For partially systematic adaptive cluster sampling, if the initial sample of a single primary sampling unit and $m-1$ secondary sampling units is selected with sampling without replacement of networks, then an unbiased estimators of the population mean and its variance (based on Salehi and Seber, 1997: 209-219 ; Dryver and Thompson, 2007: 35-43) are

$$\hat{\mu}_{networks} = \frac{1}{mNM} \sum_{i=1}^{m} Z_i^* \tag{4.34}$$

and

$$V\left(\hat{\mu}_{networks}\right) = \frac{1}{m^2 \left(NM\right)^2} \sum_{i=1}^{m} V\left(Z_i^*\right) \ , \tag{4.35}$$

where

$$\begin{aligned} Z_1 &= \frac{\sum_{j \in S_1} y_{j.} / m_j}{p_1} = \frac{\sum_{j \in S_1} \bar{y}_j}{p_1} \\ Z_2^* &= \sum_{k \in S_1^*} y_k^* + \frac{\left(1 - \sum_{k \in S_1^*} p_k^*\right) y_{2.}^*}{p_2^*} \\ &\vdots \end{aligned} \tag{4.36}$$

$$Z_m^* = \left(\sum_{k \in S_1^*} y_k^* + y_{2.}^* + \ldots + y_{(m-1).}^* \right) + \frac{\left(1 - \sum_{k \in S_1^*} p_k^* - p_2^* - \ldots - p_{m-1}^* \right) y_{m.}^*}{p_m^*}$$

Further, an unbiased estimator of $V\left(\hat{\mu}_{networks}\right)$ is given by

$$\hat{v}\left(\hat{\mu}_{networks}\right) = \frac{1}{m(m-1)} \sum_{i=1}^{m} \left(\frac{Z_i^*}{MN} - \hat{\mu}_{networks} \right)^2 \qquad (4.37)$$

Proof

To claim that $E\left(\hat{\mu}_{networks}\right) = \mu$,

$$E\left(\hat{\mu}_{networks}\right) = E\left(\sum_{i=1}^{m} Z_i^* \Big/ mNM \right) = \sum_{i=1}^{m} E\left(Z_i^*\right) \Big/ mNM \qquad (4.38)$$

Let an initial sample of size $m = 2$, means a single primary sampling unit and a secondary sampling unit, be drawn without replacement of networks. Let U denote a set of all networks in the population, i_i denote the selected unit on the i^{th} draw, $i = 1, 2, \ldots, m$, S_{i_1} denote a set of secondary sampling units within the first initial unit, $S_{i_i-1}^*$ be a set of all distinct networks included in the first-$(i-1)$ prior selection and $S_{i_i}^*$ be a set of all distinct networks included in i^{th} selection. The expectation of $\hat{\mu}_{networks}$ in this case is

$$E\left(\hat{\mu}_{networks}\right) = E\left(\frac{1}{2NM} \sum_{i=1}^{m} Z_i^* \right) = \frac{1}{2NM} \left(E\left(Z_1^*\right) + E\left(Z_2^*\right) \right) \qquad (4.39)$$

Based on the first initial unit $i_1 = i = 1$, notice that each estimator Z_i^* defined in equation (4.29) and (4.30) is a function of the sample value drawn. Since Z_1^* in (4.29) and Z_1 in (4.2) are the same formula, and by using (4.8) so we get

$$E\left(Z_1^*\right) = E_1\left(Z_i^*\right) = E_1\left\{\sum_{j \in S_{i_1}} \bar{y}_j \Big/ p_{i_1}\right\} = \sum_{i_1=1}^{N} Y_{i_1.} = \tau \tag{4.40}$$

Next, when $i = 2 = i_2$ and based on Raj (1956: 271), the expected value of Z_2^* is equal to

$$E\left(Z_2^*\right) = E_1 E_2\left(\sum_{k \in S_{i_1}^*} y_k^* + \frac{\left(1 - \sum_{k \in S_{i_1}^*} p_k^*\right) y_{i_2.}^*}{p_{i_2}^*}\right) \tag{4.41}$$

where E_1 is the expected value for all possible primary sampling units obtained in the 1st draw, E_2 is the expected value for all remaining secondary sampling units in the population excluding all secondary sampling units are in $S_{i_1}^*$.

$$= E_1\left\{E_2\left(\sum_{k \in S_{i_1}^*} y_k^* + \frac{\left(1 - \sum_{k \in S_{i_1}^*} p_k^*\right) y_{i_2}^*}{p_{i_2}^*}\middle| i_1\right)\right\}$$

$$= E_1\left\{\sum_{k \in S_{i_1}^*} y_k^* + E_2\left(\frac{\left(1 - \sum_{k \in S_{i_1}^*} p_k^*\right) y_{i_2}^*}{p_{i_2}^*}\middle| i_1\right)\right\}$$

$$= E_1\left\{\sum_{k\in S_{i_1}^*} y_k^* + \sum_{i_2\in U-S_{i_1}^*} \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right) y_{i_2}^*}{p_{i_2}^*} \Pr\left(i_2 \middle| i_1\right)\right\}$$

$$= E_1\left\{\sum_{k\in S_{i_1}^*} y_k^* + \sum_{i_2\in U-S_{i_1}^*} \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right) y_{i_2}^*}{p_i^*} \frac{p_i^*}{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)}\right\}$$

$$= E_1\left\{\sum_{k\in S_{i_1}^*} y_k^* + \sum_{i_2\in U-S_{i_1}^*} y_{i_2}^*\right\} = E_1\left\{\sum_{k\in S_{i_1}^*} y_k^* + \left(\tau - \sum_{k\in S_{i_1}^*} y_k^*\right)\right\}$$

$$= E_1\left\{\tau\right\} = \tau \text{ = population total} \qquad (4.42\ )$$

Estimator Z_2^* is an unbiased estimator of the population total so that for the general case $i = m$, we get $E\left(Z_m^*\right) = E_1E_2...E_m\left(Z_m^*\right) = \tau$. Thus the expectation of $\hat{\mu}_{networks}$ is

$$E\left(\hat{\mu}_{networks}\right) = \frac{1}{mNM}\sum_{i=1}^{m} E\left(Z_i^*\right) \qquad (4.43)$$

$$= \frac{1}{mNM}\sum_{i=1}^{m}\tau = \frac{m\tau}{mNM} = \frac{\tau}{MN} = \mu$$

It is also claimed that the estimator $\hat{\mu}_{networks}$ is an unbiased estimator of the population mean.

For the true variance of estimator $\hat{\mu}_{networks}$, using

$$V\left(\hat{\mu}_{networks}\right)=V\left(\frac{1}{NMm}\left(\sum_{i=1}^{m}Z_i^*\right)\right)=\frac{1}{\left(NM\right)^2 m^2}V\left(\sum_{i=1}^{m}Z_i^*\right) \tag{4.44}$$

$$=\frac{1}{\left(NM\right)^2 m^2}\left[\sum_{i=1}^{m}V\left(Z_i^*\right)+\sum_{i=1}^{m}\sum_{j\neq i}^{m}Cov\left(Z_i^*,Z_j^*\right)\right]$$

The selection of unit in each draw under this scheme of the selection is done independently. Based on Raj (1956: 271-273) and Hedayat and Sinha (1991: 135-136), we get the covariance between any pair of the estimators $Z_1^*, Z_2^*, ..., Z_m^*$ as zero, and $Cov\left(Z_i^*, Z_j^*\right)=0$ for all $i \neq j$. Thus the true variance of estimator $\hat{\mu}_{networks}$ will become

$$V\left(\hat{\mu}_{networks}\right)=\frac{1}{m^2\left(NM\right)^2}\sum_{i=1}^{m}V\left(Z_i^*\right) \tag{4.45}$$

Let $m=2$, the true variance of estimator $\hat{\mu}_{networks}$, be

$$V\left(\hat{\mu}_{networks}\right)=\frac{1}{m^2\left(NM\right)^2}\left\{V\left(Z_1^*\right)+V\left(Z_2^*\right)\right\}$$

For $i=1=i_1$, the variance of Z_1^* is

$$V\left(Z_1^*\right)=V\left(Z_1\right)=\left\{\sum_{i_1=1}^{N}\frac{\left(\sum_{j\in S_{i_1}}\bar{y}_j\right)^2}{p_{i_1}}\right\}-\tau^2$$

The formula of estimator Z_1^* is similar to the formula of Z_1 so the details of the proof of $V\left(Z_1^*\right)$ will be skipped.

Next, for $i = 2 = i_2$, estimator Z_2^* is also a function of the sample values drawn. Following the approach in Raj (1956: 271) and (4.21), the variance of Z_2^* is

$$V\left(Z_2^*\right) = \left(E_1V_2 + V_1E_2\right)\left(\sum_{k\in S_{i_1}^*} y_k^* + \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right) \tag{4.46}$$

where E_1 is the expected value for all possible primary sampling units obtained in the 1st draw, E_2 is the expected value for all remaining secondary sampling units in the population excluding all units within networks that are in $S_{i_1}^*$, V_1 is the variance for all possible primary sampling units obtained in the first draw and V_2 is the conditional variance for Z_2^*.

$$= E_1V_2\left(\sum_{k\in S_{i_1}^*} y_k^* + \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right) + V_1E_1\left(\sum_{k\in S_{i_1}^*} y_k^* + \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)$$

Consider in terms of $E_1V_2\left(\sum_{k\in S_{i_1}^*} y_k^* + \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)$

$$= E_1\left\{V_2\left(\sum_{k\in S_{i_1}^*} y_k^* + \frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)\right\} = E_1\left\{V_2\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)\right\}$$

$\because\ V_2\left(\sum_{k\in S_{i_1}^*} y_k^*\right)=0$ and we get

$$=E_1\left\{E_2\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}-E_2\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)\right)^2\right\}$$

Since $E_2\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)=\left(\tau-\sum_{k\in S_{i_1}^*} y_k^*\right)$ so we get

$$=E_1\left\{E_2\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)^2-2\left(\tau-\sum_{k\in S_{i_1}^*} y_k^*\right)^2+\left(\tau-\sum_{k\in S_{i_1}^*} y_k^*\right)^2\right\}$$

$$=E_1\left\{\sum_{i_2\in U-S_{i_1}^*}\left(\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)^2\Bigg|\Pr\left(i_2\middle|i_1\right)\right\}-\left(\tau-\sum_{k\in S_{i_1}^*} y_k^*\right)^2$$

$$=E_1\left\{\sum_{i_2\in U-S_{i_1}^*}\frac{\left(1-\sum_{k\in S_{i_1}^*} p_k^*\right)\left(y_{i_2}^*\right)^2}{p_{i_2}^*}\right\}-\left(\tau-\sum_{k\in S_{i_1}^*} y_k^*\right)^2$$

$$=\sum_{i_1=1}^{N}\Pr(i_1)\sum_{i_2\in U-S_{i_1}^*}\left(\frac{\left(1-\sum_{k\in S_{i_1}^*}p_k\right)\left(y_{i_2}^*\right)^2}{p_{i_2}^*}\right)-\sum_{i_1=1}^{N}\Pr(i_1)\left(\tau-\sum_{k\in S_{i_1}^*}y_k^*\right)^2$$

$$=\sum_{i_1=1}^{N}p_{i_1}\sum_{i_2\in U-S_{i_1}^*}\left(\frac{\left(1-\sum_{k\in S_{i_1}^*}p_k\right)\left(y_{i_2}^*\right)^2}{p_{i_2}^*}\right)-\sum_{i_1=1}^{N}p_{i_1}\left(\tau-\sum_{k\in S_{i_1}^*}y_k^*\right)^2$$

This gives $E_1V_2\left(Z_2^*\right)=\sum_{i_1=1}^{N}p_{i_1}\sum_{i_2\in U-S_{i_1}^*}\left(\frac{\left(1-\sum_{k\in S_{i_1}^*}p_k^*\right)\left(y_{i_2}^*\right)^2}{p_{i_2}^*}\right)-\sum_{i_1=1}^{N}p_{i_1}\left(\tau-\sum_{k\in S_{i_1}^*}y_k^*\right)^2$.

Next, consider in terms of $V_1E_2\left(Z_2^*\right)=V_1E_2\left(\sum_{k\in S_{i_1}^*}y_k^*+\frac{\left(1-\sum_{k\in S_{i_1}^*}p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)$

$$=V_1\left\{E_2\left(\sum_{k\in S_{i_1}^*}y_k^*+\frac{\left(1-\sum_{k\in S_{i_1}^*}p_k^*\right)y_{i_2}^*}{p_{i_2}^*}\right)\right\}$$

$$=V_1\left\{\sum_{k\in S^*_{i_1}} y^*_k + E_2\left(\frac{\left(1-\sum_{k\in S^*_{i_1}} p^*_k\right)y^*_{i_2}}{p^*_{i_2}}\right)\right\}$$

$$=V_1\left\{\sum_{k\in S^*_{i_1}} y^*_k + \sum_{i_2\in U-S^*_{i_1}} \frac{\left(1-\sum_{k\in S^*_{i_1}} p^*_k\right)y^*_{i_2}}{p^*_i}\Pr\left(i_2 \middle| i_1\right)\right\}$$

$$=V_1\left\{\sum_{k\in S^*_{i_1}} y^*_k + \sum_{i\in U-S^*_{i_1}} y^*_{i_1.}\right\} = V_1\left\{\sum_{k\in S^*_{i_1}} y^*_k + \left(\tau - \sum_{k\in S^*_{i_1}} y^*_k\right)\right\} = V_1\{\tau\} = 0$$

So that we get the true variance of Z^*_2 as

$$V\left(Z^*_2\right) = \sum_{i_1=1}^{N} p_{i_1} \sum_{i_2\in U-S^*_{i_1}} \left(\frac{\left(1-\sum_{k\in S^*_{i_1}} p^*_k\right)\left(y^*_{i_2.}\right)^2}{p^*_{i_1}}\right) - \sum_{i_1=1}^{N} p_{i_1}\left(\tau - \sum_{k\in S^*_{i_1}} y^*_k\right)^2, \qquad (4.47)$$

where i_1 and i_2 are units obtained in the 1^{st} and the 2^{nd} draws.

Thus, the true variance of Z^*_i for general $i = m = i_m$ is

$$V\left(Z_m^*\right)=\sum_{i_1=1}^{N} p_{i_1} \sum_{i_2 \in U-S_{i_1}^*} p_{i_2}^* \cdots \sum_{i_m \in U-S_{i_1}^*-\ldots-S_{i_{m-1}}^*} \left(\frac{\left(1-\sum_{k \in S_{i_1}^*} p_k^* - p_{i_2}^* \ldots - p_{i_{m-1}}^*\right)\left(y_{i_m \cdot}^*\right)^2}{p_{i_m}^*} \right) \tag{4.48}$$

$$-\sum_{i_1=1}^{N} p_{i_1} \sum_{i_2 \in U-S_{i_1}^*} p_{i_2}^* \cdots \sum_{i_m \in U-S_{i_1}^*-\ldots-S_{i_{m-1}}^*} \left(\tau - \sum_{k \in S_{i_1}^*} y_k^* - y_{i_2 \cdot}^* \ldots - y_{i_{m-1}}^*\right)^2$$

Next, to show that $\hat{v}\left(\hat{\mu}_{networks}\right)$ is an unbiased estimator of $V\left(\hat{\mu}_{networks}\right)$

$$E\left(\hat{v}\left(\hat{\mu}_{networks}\right)\right)=E\left\{\frac{1}{m(m-1)}\sum_{i=1}^{m}\left(\frac{Z_i^*}{NM}-\hat{\mu}_{networks}\right)^2\right\} \tag{4.49}$$

$$=E\left\{\frac{1}{m(m-1)}\sum_{i=1}^{m}\left[\left(\frac{Z_i^*}{NM}\right)^2-2\left(\frac{Z_i^*}{NM}\right)\hat{\mu}_{networks}+\hat{\mu}_{networks}^2\right]\right\}$$

$$=E\left\{\frac{1}{m(m-1)}\left[\sum_{i=1}^{m}\left(\frac{Z_i^*}{NM}\right)^2-2\sum_{i=1}^{m}\left(\frac{Z_i^*}{NM}\right)\hat{\mu}_{networks}+\sum_{i=1}^{m}\hat{\mu}_{networks}^2\right]\right\}$$

$$=E\left\{\frac{1}{m(m-1)}\left\{\sum_{i=1}^{m}\frac{Z_i^{*2}}{(MN)^2}-2m\hat{\mu}_{networks}^2+m\hat{\mu}_{networks}^2\right\}\right\}$$

$$=E\left\{\frac{\sum_{i=1}^{m}Z_i^{*2}}{m(m-1)(MN)^2}-\frac{m\hat{\mu}_{networks}^2}{m(m-1)}\right\}$$

Since $\frac{m\hat{\mu}_{networks}}{m(m-1)}=\frac{1}{(m-1)}\left(\frac{\sum_{i=1}^{m}Z_i^*}{mMN}\right)^2=\frac{1}{m-1}\left(\frac{\sum_{i=1}^{m}Z_i^{*2}+2\sum_{i=1}^{m}\sum_{i<j}^{m}Z_i^*Z_j^*}{m^2(NM)^2}\right)$ we get

$$= E\left\{\frac{\sum_{i=1}^{m} Z_i^{*2}}{m(m-1)(MN)^2} - \frac{m\hat{\mu}_{networks}^2}{m(m-1)}\right\}$$

$$= \left\{\frac{\sum_{i=1}^{m} E\left(Z_i^{*2}\right)}{m(m-1)(MN)^2}\right\} - \left\{\frac{E\left(\sum_{i=1}^{m} Z_i^{*2} + 2\sum_{i=1}^{m}\sum_{i<j}^{m} Z_i^* Z_j^*\right)}{m^2(m-1)(MN)^2}\right\}$$

$$= \left\{\frac{\sum_{i=1}^{m} E\left(Z_i^{*2}\right)}{m(m-1)(MN)^2}\right\} - \left\{\frac{\left(\sum_{i=1}^{m} E\left(Z_i^{*2}\right) + 2\sum_{i=1}^{m}\sum_{i<j}^{m} E\left(Z_i^* Z_j^*\right)\right)}{m^2(m-1)(MN)^2}\right\}$$

$$= \left\{\frac{\sum_{i=1}^{m}\left(V\left(Z_i^*\right) + \left(E\left(Z_i^*\right)\right)^2\right)}{m(m-1)(MN)^2}\right\} - \left\{\frac{\sum_{i=1}^{m}\left(V\left(Z_i^*\right) + \left(E\left(Z_i^*\right)\right)^2\right)}{m^2(m-1)(MN)^2}\right\}$$

$$- \left\{\frac{2\sum_{i=1}^{m}\sum_{i<j}^{m}\left(Cov\left(Z_i^*, Z_j^*\right) - E\left(Z_i^*\right)E\left(Z_j^*\right)\right)}{m^2(m-1)(MN)^2}\right\}$$

Since $E\left(Z_i^*\right) = \tau$, $E\left(Z_i^*\right)E\left(Z_j^*\right) = \tau^2$ and $Cov\left(Z_i^*, Z_j^*\right) = 0$, we get

$$= \left\{\frac{\sum_{i=1}^{m}\left(V\left(Z_i^*\right) + \tau^2\right)}{m(m-1)(MN)^2}\right\} - \left\{\frac{\left(\sum_{i=1}^{m}\left(V\left(Z_i^*\right) + \tau^2\right) - 2\sum_{i=1}^{m}\sum_{i<j}^{m}\tau^2\right)}{m^2(m-1)(MN)^2}\right\}$$

$$= \left\{ \frac{\sum_{i=1}^{m} V\left(Z_i^*\right)}{m(m-1)(MN)^2} - \frac{\sum_{i=1}^{m} V\left(Z_i^*\right)}{m^2(m-1)(MN)^2} \right\}$$

$$+ \left\{ \frac{m\tau^2}{m(m-1)(MN)^2} - \frac{m\tau^2}{m^2(m-1)(MN)^2} - \frac{2\left(m(m-1)/2\right)\tau^2}{m^2(m-1)(MN)^2} \right\}$$

$$= \left\{ \frac{(m-1)\sum_{i=1}^{m} V\left(Z_i^*\right)}{(MN)^2 m^2 (m-1)} \right\} + \left\{ \frac{m^2\tau^2 - m\tau^2 - \left(m(m-1)\tau^2\right)}{(MN)^2 m^2 (m-1)} \right\}$$

$\because \left\{ \frac{m^2\tau^2 - m\tau^2 - \left(m(m-1)\tau^2\right)}{(MN)^2 m^2 (m-1)} \right\} = 0$, thus

$$E\left(\hat{v}\left(\hat{\mu}_{networks}\right)\right) = \left\{ \frac{1}{(MN)^2 m^2} \sum_{i=1}^{m} V\left(Z_i^*\right) \right\} = V\left(\hat{\mu}_{networks}\right) \tag{4.50}$$

This means that the estimator $\hat{v}\left(\hat{\mu}_{networks}\right)$ is an unbiased estimator of $V\left(\hat{\mu}_{networks}\right)$.

4.3.4 When the Design is a Selection Without Replacement of Clusters

The first-draw probability for each primary unit, that is $p_1 = 1/N$, is not changed. For $i = 1$, let

$$Z_i'' = Z_i = \frac{\sum_{j \in S_1} y_{j.} / m_j}{p_i} = \frac{\sum_{j \in S_1} \bar{y}_j}{p_i} \tag{4.51}$$

For $i = 2,3,...,m$, the relevant probabilities $p_i^* = m_{i.}/MN$ are not changed and the conditional probability for each secondary unit in the i^{th} draw given the first (i-1) prior draw is $p_i^* \Big/ 1 - \sum_{k \in S''_{i-1}} p_k^*$. Let

$$Z_i'' = \sum_{k \in S''_{i-1}} y_k'' + \frac{\left(1 - \sum_{k \in S''_{i-1}} p_k^*\right) y_{i.}^*}{p_i^*} \tag{4.52}$$

where S''_{i-1} is a group of all distinct secondary sampling units included in the first (i-1) prior draw, $y_{i.}^*$ is the total y-values of network including the i^{th} unit of the sample. When $i-1=1$, $S''_{i-1} = S''_1$ represents a group of all distinct secondary sampling units included in the 1^{st} draw and S''_1 equals S_1^* plus their associated edge units obtained in the 1^{st} draw, $y''_{k.}$ is the total y-values of distinct secondary sampling units within a cluster associated with network k in S''_{i-1}. When $i-1 \geq 2$, notice that the number of networks obtained in draw i is only one network so that S''_{i-1} contains all distinct secondary sampling units corresponding to network k which obtained in the first (i-1) prior draw and $\sum_{k \in S''_{i-1}} p_k^* = p''_{i-1}$ and $\sum_{k \in S^*_{i-1}} y_k^* = y''_{i-1.}$.

Theorem 4.3 For partially systematic adaptive cluster sampling, if the initial sample of a single primary sampling unit and $m-1$ secondary sampling units is selected with sampling without replacement of clusters, then an unbiased estimator of the population mean (based on Dryver and Thompson, 2007: 35-43) is

$$\hat{\mu}_{clusters} = \frac{1}{mNM} \sum_{i=1}^{m} Z_i'' \tag{4.53}$$

with variance

$$V\left(\hat{\mu}_{clusters}\right) = \frac{1}{m^2 \left(NM\right)^2} \sum_{i=1}^{m} V\left(Z_i''\right), \tag{4.54}$$

where

$$Z_1 = \frac{\sum_{j \in S_1} y_{j.} / m_j}{p_1} = \frac{\sum_{j \in S_1} \bar{y}_j}{p_1}$$

$$Z_2'' = \sum_{k \in S_1''} y_k'' + \frac{\left(1 - \sum_{k \in S_1''} p_k^*\right) y_{2.}^*}{p_2^*} \tag{4.55}$$

$$\vdots$$

$$Z_m'' = \left(\sum_{k \in S_1''} y_{k.}'' + y_{2.}'' + \ldots + y_{m-1.}'' \right) + \frac{\left(1 - \sum_{k \in S_1''} p_k^* - p_2'' - \ldots - p_{m-1}''\right) y_{m.}''}{p_m^*}$$

and an unbiased estimator of $V\left(\hat{\mu}_{clusters}\right)$ is

$$\hat{v}\left(\hat{\mu}_{clusters}\right) = \frac{1}{m(m-1)} \sum_{i=1}^{m} \left(\frac{Z_i''}{MN} - \hat{\mu}_{clusters} \right)^2 \tag{4.56}$$

The proof can be done in the same way as proving theorem 4.1 and theorem 4.2.

4.4 Simulation Study

4.4.1 An Illustrative Example

The calculation and properties of the proposed estimators in section 3 and estimators proposed by Raj (1956: 296-284) for the population mean and the variance is described in this section. The population consists of 12 secondary sampling units

(SSUs) with giving a population mean of μ = 2.25 and population variance of σ^2 = 8.02. In table 4.1, the details of the individual values of population is the number appears in cell $i-j$. The condition for adding units is defined by $C=\left\{Y_{ij}>2\right\}$. So that there are total 10 networks in this population. The corresponding value of the networks and clusters, which is sometimes called the transformed population, is shown in table 4.1. A primary sampling unit (PSU) is defined as a set of secondary sampling units (SSUs) that are arranged in horizontal strip pattern. The initial sample of size 2 units, a single PSU and a SSU, is considered. So that the first initial unit starts with the selection of a starting point from the 3-by-1 squares in table 4.1 and all SSUs with the same positions are formed the selected PSU. The first draw probability of choosing a single PSU is $p_1 = 1/3$.

Table 4.1 A Population Consists of $N=3$ PSUs and Its Associated Network with the Number of PSU Corresponding with the Network.

Population; y_{ij}				Corresponding Network Value; $y_{k.}^{*}$				Corresponding Cluster Value; $y_{k.}''$			
5	10	1	0	5+10=15 *(1)*	15 *(1)*	1 *(2)*	0 *(3)*	5+10+1+2 =18	18	1	0
2	1	0	3	2 *(4)*	1 *(5)*	0 *(6)*	3+4=7 *(7)*	2	1	0	3+4+1+0 =8
0	0	1	4	0 *(8)*	0 *(9)*	1 *(10)*	7 *(7)*	0	0	1	8

Corresponding network size; $m_{k.}$				Corresponding cluster size; $m_{k.}^{*}$			
2	2	1	1	4	4	1	1
1	1	1	2	1	1	1	4
1	1	1	2	1	1	1	4

Note: The number in the parenthesis represents the network labels.

Based on information in table 4.1, it will be shown how to calculate the proposed estimators of the population mean and variance when a selection is without replacement of units, without replacement of networks and without replacement of clusters, respectively. Suppose that the 2nd primary sampling unit (PSU) is selected as the 1st initial unit with giving an individual y-values {2, 1, 0, 3} and network values {2, 1, 0, 7}. Notice that the 1st initial unit gives 4 distinct networks because secondary sampling units (SSUs) within it belong to the 4th, 5th, 6th and 7th network.

When a selection is without replacement of units, we get $S_1 = \{ j : 1 \le j \le M = 4 \}$, where j represents secondary sampling unit label, and values of $y_{j.}$ and $m_{j.}$ are {2, 1, 0, 7} and {1, 1, 1, 2}, respectively. So that we get

$$Z_1 = \sum_{j \in S_1} \left(y_{j.} / m_{j.} \right) \Big/ p_1 = \sum_{j \in S_1} \bar{y}_j \Big/ p_1 = \left(2 + 1 + 0 + 3.5 \right) / \left(1/3 \right) = 19.50 \qquad (4.57)$$

For the second initial unit or $i = 2$, it can be chosen from total of 8 secondary sampling units remaining in the population by excluding all of 4 secondary sampling units in the 1st initial unit. Suppose that unit (1, 1) is chosen in the 2nd draw with giving y-values {5; 10, 1}, where additional units are appearing after the semicolon. The conditional probability of choosing this unit given the first initial unit has been selected is

$$p_2' \Big/ \left(1 - \sum_{j \in S_1'} p_j' \right) = \left(1/12 \right) \Big/ \left(1 - \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \right) \right) = \frac{1}{8} \text{ , and we get}$$

$$\begin{aligned} Z_2 &= \sum_{j \in S_1'} \bar{y}_j + \left[\left(1 - \sum_{j \in S_1'} p_j' \right) \bar{y}_2 \Big/ p_2' \right] = \left(2 + 1 + 0 + 3.5 \right) + \left(\frac{7.5}{1/8} \right) \\ &= 66.5 \end{aligned} \qquad (4.58)$$

Also, the estimated of the population mean and its variance is

$$\hat{\mu}_{units} = \left(19.50 + 66.5 \right) / \left(12 \left(2 \right) \right) = 3.58 \qquad (4.59)$$

and

$$\begin{aligned} \hat{v}\left(\hat{\mu}_{units} \right) &= \left\{ \left(\left(\frac{19.50}{12} \right) - 3.583 \right)^2 + \left(\left(\frac{66.5}{12} \right) - 3.583 \right)^2 \right\} \Big/ \left(2 \left(2 - 1 \right) \right) \\ &= 3.835 \end{aligned} \qquad (4.60)$$

For a selected without replacement of networks, because the 1st initial unit gives 4 distinct networks, these are the 4th, 5th, 6th and 7th network, we get $S_1^* = \{k : 4 \le k \le 7\}$, where k represents network label, and the calculation of Z_1^* is

$$Z_1^* = Z_1 = \sum_{j \in S_1} \bar{y}_j \Big/ p_1 = (2 + 1 + 0 + 3.5) \big/ (1/3) = 19.50 \tag{4.61}$$

Based on a selection is without replacement of networks, the second initial unit can be chosen from total of 5 networks remaining in the population by excluding all of 4 networks have been selected in the first draw. If a unit (1, 1) is chosen then its network values is {15; 1}, where additional unit is appearing after the semicolon. Notice that the network including unit (1, 1), that is the 1st network, is also selected in this draw. So that we get $y_{2\square} = 15$ and $m_{2\square} = 2$. The conditional probability of choosing this network given all of 4 networks associated with the 1st initial unit have been chosen is

$$p_2^* \Big/ \left(1 - \sum_{j \in S_1^*} p_k^*\right) = (2/12) \Big/ \left(1 - \left(\frac{1}{12} + \frac{1}{12} + \frac{2}{12} + \frac{1}{12}\right)\right) = \frac{2}{7},$$

and the calculation of Z_2^* is

$$Z_2^* = \sum_{k \in S_1^*} y_k + \left[\left(1 - \sum_{k \in S_1^*} p_k^*\right) y_{2\square} \Big/ p_2^*\right] = (2 + 1 + 7 + 0) + \left(\frac{15}{2/7}\right)$$
$$= 62.50 \tag{4.62}$$

The estimated of the population mean and its variance is

$$\hat{\mu}_{networks} = (19.5 + 62.50) \big/ (12(2)) = 3.42 \tag{4.63}$$

and

$$\hat{v}(\hat{\mu}_{networks}) = \left\{\left(\left(\frac{19.5}{12}\right) - 3.42\right)^2 + \left(\left(\frac{62.50}{12}\right) - 3.42\right)^2\right\} \Big/ (2(2-1))$$
$$= 3.210 \tag{4.64}$$

For sampling without replacement of clusters, the 1st initial unit gives network values {2, 1, 0, 7; 0, 1}, where additional units are appearing after the semicolon. This means that the 4th, 5th, 6th, 7th; 3rd and 10th network is also selected in the first draw. But Z_1'' is calculated by

$$Z_1'' = Z_1 = \sum_{j \in S_1} \bar{y}_j \Big/ p_1 = (2+1+0+3.5)\big/(1/3) = 19.50 \tag{4.65}$$

The second initial unit can be chosen from total of 4 networks remaining in the population by excluding all of the distinct secondary sampling units that are included in the sample in the first draw. When a unit (1, 1) with network values of {15; 1}, where additional unit is appearing after the semicolon, is chosen in the 2nd draw, we get $y_{2\square} = 15$ and $m_{2\square} = 2$. The conditional probability of choosing this network given all distinct units included in the 1st initial unit have been chosen is

$$p_2^* \Big/ \left(1 - \sum_{j \in S_1''} p_k^*\right) = (2/12) \Big/ \left(1 - \left(\frac{1}{12} + \frac{1}{12} + \frac{4}{12} + \frac{1}{12}\right)\right) = 0.400,$$

and the calculation of Z_2'' is

$$\begin{aligned} Z_2'' &= \sum_{k \in S_1''} y_k'' + \left[\left(1 - \sum_{k \in S_1''} p_k^*\right) y_{2\square}'' \Big/ p_2^*\right] = (2+1+0+7+0+1) + \left(\frac{15}{\;}\right) \\ &= 48.5 \end{aligned} \tag{4.66}$$

The estimated of the population mean and its variance is

$$\hat{\mu}_{clusters} = (19.5 + 48.5)\big/(12(2)) = 2.83 \tag{4.67}$$

and

$$\begin{aligned} v(\hat{\mu}_{clusters}) &= \left\{\left(\left(\frac{19.5}{12}\right) - 2.83\right)^2 + \left(\left(\frac{48.5}{12}\right) - 2.83\right)^2\right\} \Big/ (2(2-1)) \\ &= 1.460 \end{aligned} \tag{4.68}$$

The lists of all possible adaptive cluster samples with units selected without replacement and the corresponding estimators $\hat{\mu}_{units}$ and $\hat{v}\left(\hat{\mu}_{units}\right)$ are shown in table 4.2. The result show that both estimators $\hat{\mu}_{units}$ and $\hat{v}\left(\hat{\mu}_{units}\right)$ are unbiased estimator of the population mean and the variance. Lists of all possible adaptive cluster samples with networks selected without replacement and clusters selected without replacement are shown in table 4.3 and table 4.4. The result show that both estimators $\hat{\mu}_{networks}$, and $\hat{\mu}_{clusters}$ are unbiased estimator of the population mean (see table 4.3 and 4.4). Similar to the expectation of estimators $\hat{v}\left(\hat{\mu}_{networks}\right)$ and $\hat{v}\left(\hat{\mu}_{clusters}\right)$ indicate that both estimators are unbiased estimator of the variance. In addition, the estimator $\hat{\mu}_{clusters}$ gives the smallest variance while the estimator $\hat{\mu}_{units}$ gives the largest variance.

Table 4.2 All Possible Adaptive Cluster Samples with Units Selected Without Replacement for Small Population and the Corresponding Estimators $\hat{\mu}_{units}$ and $\hat{v}(\hat{\mu}_{units})$.

Observed units *	**Sampling w/o of units**			
	$p(s)$	ν	$\hat{\mu}_{units}$	$\hat{v}(\hat{\mu}_{units})$
(5, 10, 1, 0, 2; 1)	1/24	6	3.33	0.444
(5, 10, 1, 0, 1; 2)	1/24	6	3.00	1.000
(5, 10, 1, 0, 0; 2,1)	1/24	7	2.67	1.778
(5, 10, 1, 0, 3; 4, 1, 2, 1, 0)	1/24	10	2.83	0.028
(5, 10, 1, 0, 0; 2, 1)	1/24	7	2.67	1.778
(5, 10, 1, 0, 0; 2, 1)	1/24	7	2.67	1.778
(5, 10, 1, 0, 1; 2, 1)	1/24	7	3.00	1.000
(5, 10, 1, 0, 4; 3, 1, 0, 2, 1)	1/24	10	3.83	0.028
(2, 1, 0, 3, 5; 10, 1, 4, 1, 0)	1/24	10	3.58	3.835
(2, 1, 0, 3, 10; 5, 1, 4, 1, 0)	1/24	10	3.58	3.835
(2, 1, 0, 3, 1; 0, 4, 1)	1/24	8	1.42	0.043
(2, 1, 0, 3, 0; 0, 4, 1)	1/24	8	1.08	0.293
(2, 1, 0, 3, 0; 0, 4, 1)	1/24	8	1.08	0.293
(2, 1, 0, 3, 0; 0, 4, 1)	1/24	8	1.08	0.293
(2, 1, 0, 3, 1; 0, 4, 1)	1/24	8	1.42	0.043
(2, 1, 0, 3, 4; 0, 1)	1/24	7	2.25	0.391
(0, 0, 1, 4, 5; 3, 0, 0, 10, 1, 2, 1)	1/24	12	3.25	4.516
(0, 0, 1, 4, 10; 3, 0, 0, 5, 1, 2, 1)	1/24	12	3.25	4.516
(0, 0, 1, 4, 1; 3, 0, 0)	1/24	8	1.08	0.002
(0, 0, 1, 4, 0; 3, 0)	1/24	7	0.75	0.141
(0, 0, 1, 4, 2; 3, 0, 0)	1/24	8	1.42	0.085
(0, 0, 1, 4, 1; 3, 0, 0)	1/24	8	1.08	0.002
(0, 0, 1, 4, 0; 3, 0)	1/24	7	0.75	0.141
(0, 0, 1, 4, 3; 0, 0)	1/24	7	1.92	0.627
Mean			**2.25**	**1.120**
Variance			**1.1204**	**-**

Note: ***** considered in terms of units selected and adaptively added units are appearing after the semicolon, w/o refers to without replacement and ν refers to the final sample size.

Table 4.3 All Possible Adaptive Cluster Samples with Networks Selected Without Replacement for Small Population and the Corresponding Estimators $\hat{\mu}_{networks}$ and $\hat{v}(\hat{\mu}_{networks})$.

Observed networks *	Sampling w/o of networks			
	$p(s)$	ν	$\hat{\mu}_{networks}$	$\hat{v}(\hat{\mu}_{networks})$
(15, 1, 0, 2; 1)	1/24	6	3.33	0.444
(15, 1, 0, 1; 2)	1/24	6	3.00	1.000
(15, 1, 0, 0; 2, 1)	1/24	7	2.67	1.778
(15, 1, 0, 7; 2, 1,0,1)	2/24	10	3.83	0.028
(15, 1, 0, 0; 2, 1)	1/24	7	2.67	1.778
(15, 1, 0, 0; 2, 1)	1/24	7	2.67	1.778
(15, 1, 0, 1; 2, 1)	1/24	7	3.00	1.000
(2, 1, 0, 7, 15; 0, 1, 1)	2/21	10	3.42	3.210
(2, 1, 0, 7, 1; 0, 1)	1/24	8	1.52	0.011
(2, 1, 0, 7, 0; 0, 1)	1/24	8	1.23	0.157
(2, 1, 0, 7, 0; 0, 1)	1/24	8	1.23	0.157
(2, 1, 0, 7, 0; 0, 1)	1/24	8	1.23	0.157
(2, 1, 0, 7, 1; 0)	1/24	7	1.52	0.011
(2, 1, 0, 7, 7; 0, 0, 1)	-	-	-	-
(0, 0, 1, 7, 15; 0, 0, 1, 2, 1)	2/21	12	3.08	3.835
(0, 0, 1, 7, 1; 0, 0)	1/24	8	1.19	0.004
(0, 0, 1, 7, 0; 0)	1/24	7	0.90	0.053
(0, 0, 1, 7, 2; 0, 0)	1/24	8	1.48	0.125
(0, 0, 1, 7, 1; 0, 0)	1/24	8	1.19	0.004
(0, 0, 1, 7, 0; 0)	1/24	7	0.90	0.053
(0, 0, 1, 7, 7; 0, 0)	-	-	-	-
Expected value			**2.25**	**1.032**
Variance			**1.032**	**-**

Note: ***** considered in terms of networks selected and adaptively added units are appearing after the semicolon, w/o refers to without replacement and ν refers to the final sample size.

Table 4.4 All Possible Adaptive Cluster Samples with Clusters Selected Without Replacement for Small Population and the Corresponding Estimators $\hat{\mu}_{clusters}$ and $\hat{v}(\hat{\mu}_{clusters})$.

Observed networks *	**Sampling w/o of clusters**			
	$p(s)$	ν	$\hat{\mu}_{clusters}$	$\hat{v}(\hat{\mu}_{clusters})$
(15, 1, 0, 2; 1)	0	6	-	-
(15, 1, 0, 1; 2)	0	6	-	-
(15, 1, 0, 0; 2, 1)	1/18	7	2.79	1.460
(15, 1, 0, 7; 2, 1,0,1)	2/18	10	3.67	0.111
(15, 1, 0, 0; 2, 1)	1/18	7	2.79	1.460
(15, 1, 0, 0; 2, 1)	1/18	7	2.79	1.460
(15, 1, 0, 1; 2, 1)	1/18	7	3.04	0.918
(2, 1, 0, 7, 15; 0, 1, 1)	2/15	10	2.83	1.460
(2, 1, 0, 7, 1; 0, 1)	1/15	8	1.48	0.021
(2, 1, 0, 7, 0; 0, 1)	-	-	-	-
(2, 1, 0, 7, 0; 0, 1)	1/15	8	1.27	0.125
(2, 1, 0, 7, 0; 0, 1)	1/15	8	1.27	0.125
(2, 1, 0, 7, 1; 0)	-	-	-	-
(2, 1, 0, 7, 7; 0, 0, 1)	-	-	-	-
(0, 0, 1, 7, 15; 0, 0, 1, 2, 1)	2/15	12	2.46	1.778
(0, 0, 1, 7, 1; 0, 0)	1/15	8	1.10	0.0004
(0, 0, 1, 7, 0; 0)	-	-	-	-
(0, 0, 1, 7, 2; 0, 0)	1/15	8	1.31	0.035
(0, 0, 1, 7, 1; 0, 0)	1/15	8	1.10	0.0004
(0, 0, 1, 7, 0; 0)	-	-	-	-
(0, 0, 1, 7, 7; 0, 0)	-	-	-	-
Expected value			**2.25**	**0.759**
Variance			**0.759**	**-**

Note: ***** considered in terms of networks selected and adaptively added units are appearing after the semicolon, w/o refers to without replacement and ν refers to the final sample size.

4.4.2 Simulation Study

In order to investigate the properties of the proposed estimators, the simulation study has been carried out with real blue-winged teal data which was given by Smith et al. (1995: 777-778). The population consists of $NM = 200$ secondary sampling units (SSUs) with giving the population mean of μ = 70.605 and population variance of σ^2 = 451,440.97. These real data are shown in figure 4.2. In figure 4.2, the number appearing in cell $i-j$ is the value of Y_{ij} associated with unit (i, j). This population is partition into $N = 10$ primary sampling units (PSUs) of size $M = 20$ secondary sampling units (SSUs) and $N = 5$ primary sampling units (PSUs) of size $M = 40$ secondary sampling units (SSUs), respectively. The condition for adding units is defined by $C = \{Y_{ij} \geq 1\}$. The initial sample of size m which is composed of a single PSU and $m-1$ SSUs, where m = 2, 3, 4, 5, 6 is considered in this study. The sampling of these m initial units with a selection is without replacement of units, without replacement of networks and without replacement of clusters is carried out using the Visual-Basic package and replicated $r = 30,000$ times. The first draw probability of choosing a single PSU is $p_1 = 1/N$.

0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	20	4	2	12	0	0	0	0	0	10	103	0	0	0
0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	150	7144	1	0
0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0	6	6339	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14	122	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	114	60
0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	3	0

Figure 4.2 Real Blue-Winged Teal Data with μ = 70.605 and σ^2 = 451,440.97.

The aim of simulation study is to make a comparison between proposed estimators which are appearing in section 3 and non-adaptive estimator proposed by Raj in 1956 in terms of efficiency. Let subscript i represents the number of iterations and subscripts $(\bullet)$ be replaced with "unit," "net," "clus" and "raj" for sampling without replacement of units, sampling without replacement of networks, sampling without replacement of clusters and sampling without replacement of units with non-adaptive estimator proposed by Raj (1956: 269-284), respectively.

The simulation mean of $\hat{\mu}_{(\bullet)}$ is done based on the formula

$$E\left[\hat{\mu}_{(\bullet)}\right] = \overline{\mu}_{(\bullet)} = \frac{1}{30,000}\sum_{i=1}^{30,000}\hat{\mu}_{(\bullet)i} \qquad , \tag{4.69}$$

where $\hat{\mu}_{(\bullet)i}$ is the estimated of the population mean obtained from the i^{th} iteration.

The simulation variance of $\hat{\mu}_{(\bullet)}$ is done based on the formula

$$v\left(\hat{\mu}_{(\bullet)}\right) = \frac{1}{30,000-1}\sum_{i=1}^{30,000}\left(\hat{\mu}_{(\bullet)i} - \overline{\mu}_{(\bullet)}\right)^2 , \tag{4.70}$$

where $\overline{\mu}_{(\bullet)}$ is as in equation 4.69.

The simulation mean of $\hat{v}\left(\hat{\mu}_{(\bullet)}\right)$ is based on the formula

$$E\left[\hat{v}\left(\hat{\mu}_{(\bullet)}\right)\right] = \overline{v}\left(\hat{\mu}_{(\bullet)}\right) = \frac{1}{30,000}\sum_{i=1}^{30,000}\hat{v}\left(\hat{\mu}_{(\bullet)i}\right) \tag{4.71}$$

For the ability to construct the confidence interval, define $u_i\left(\hat{\mu}_{(\bullet)}\right)$ to be 1 if the 95% confidence interval that is produced from $\hat{\mu}_{(\bullet)i} \pm t_{\frac{\alpha}{2},m-1}\sqrt{\hat{v}\left(\hat{\mu}_{(\bullet)i}\right)}$ contains the population mean, and 0 if the 95% confidence interval does not contain the population mean. The confidence level is calculated based on

$$CI\left(\hat{\mu}_{(\bullet)}\right)=\frac{1}{30,000}\sum_{i=1}^{30,000}u_i\left(\hat{\mu}_{(\bullet)}\right) \tag{4.72}$$

The results of sampling of m initial units with units, networks and clusters without replacement and efficiency comparison results are as follows:

The simulation mean and variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and non-adaptive estimator $\hat{\mu}_{raj}$ is shown in table 4.5 and table 4.7 for the population consists of $N=10$ and $N=5$ primary sampling units. For $N=10$ primary sampling units, as seen in table 4.6, the 95% confidence interval that is formed by using the simulation mean and variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and non-adaptive estimator $\hat{\mu}_{raj}$ in any m will include the population mean. Similar to the 95% confidence interval of μ that is formed by using the simulation mean and variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and non-adaptive estimator $\hat{\mu}_{raj}$ in cases of $N=5$ and any m will include the population mean (see table 4.8).

As seen in table 4.9 and table 4.10, the simulation variance of estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and non-adaptive estimator $\hat{\mu}_{raj}$ will decrease when the initial sample size increased for both cases of $N=10$ and $N=5$ PSUs. When the proposed estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ are compared to each other, the estimator $\hat{\mu}_{clus}$ seems to be the best choice in terms of minimum variance. The estimator $\hat{\mu}_{clus}$ is the most efficient of the four estimators and its efficiency gain over non-adaptive estimator $\hat{\mu}_{raj}$ increases with m (see table 4.9 and table 4.10). In addition, the efficiency of estimators $\hat{\mu}_{unit}$ and $\hat{\mu}_{net}$ gain over non-adaptive estimator $\hat{\mu}_{raj}$ when m is increased.

The simulation means of estimators $\hat{v}\left(\hat{\mu}_{unit}\right)$, $\hat{v}\left(\hat{\mu}_{net}\right)$, $\hat{v}\left(\hat{\mu}_{clus}\right)$ and $\hat{v}\left(\hat{\mu}_{raj}\right)$ is shown in table 4.11 and table 4.12 for cases of $N=10$ and $N=5$ primary sampling units, respectively. The results show that their values are close to the estimated true variance in table 4.9 and table 4.10 This means that all of estimators $\hat{v}\left(\hat{\mu}_{unit}\right)$, $\hat{v}\left(\hat{\mu}_{net}\right)$, $\hat{v}\left(\hat{\mu}_{clus}\right)$ and $\hat{v}\left(\hat{\mu}_{raj}\right)$ are unbiased estimator of $v\left(\hat{\mu}_{unit}\right)$, $v\left(\hat{\mu}_{net}\right)$, $v\left(\hat{\mu}_{clus}\right)$ and $v\left(\hat{\mu}_{raj}\right)$, respectively.

Table 4.5 The Simulation Mean and Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and $\hat{\mu}_{raj}$ for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

m	$\bar{\mu}_{unit}$	$\bar{\mu}_{net}$	$\bar{\mu}_{clus}$	$\bar{\mu}_{raj}$	$v(\hat{\mu}_{unit})$	$v(\hat{\mu}_{net})$	$v(\hat{\mu}_{clus})$	$v(\hat{\mu}_{raj})$
1 PSU+ 1 SSU	72.19	70.11	69.04	72.79	30,314.75	22,264.45	21,041.99	97,970.30
1 PSU+ 2 SSUs	71.97	72.18	69.19	72.01	24,891.37	19,165.14	16,962.59	83,024.61
1 PSU+ 3 SSUs	69.40	70.23	69.74	70.19	19,606.92	15,749.96	14,452.13	66,727.47
1 PSU+ 4 SSUs	71.38	69.32	69.58	69.94	16,871.81	13,616.20	12,026.51	57,676.74
1 PSU+ 5 SSUs	70.48	71.32	71.59	69.73	14,507.58	11,911.05	10,209.43	49,309.03

Table 4.6 95% CI of μ That is Produced by the Simulation Mean and Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and $\hat{\mu}_{raj}$ When the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

m	$\bar{\mu}_{unit} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{unit})}{r}}$	$\bar{\mu}_{net} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{net})}{r}}$	$\bar{\mu}_{clus} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{clus})}{r}}$	$\bar{\mu}_{raj} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{raj})}{r}}$
1 PSU+ 1 SSU	(70.22, 74.15)	(68.42, 71.79)	(67.40, 70.68)	(69.25, 76.33)
1 PSU+ 2 SSUs	(70.19, 73.75)	(70.61, 73.74)	(67.72, 70.67)	(68.75, 75.27)
1 PSU+ 3 SSUs	(67.81, 70.97)	(68.81, 71.65)	(68.38, 71.09)	(67.26, 73.10)
1 PSU+ 4 SSUs	(69.91, 72.84)	(68.00, 70.64)	(68.33, 70.81)	(67.23, 72.66)
1 PSU+ 5 SSUs	(69.12, 71.84)	(70.08, 72.55)	(70.45, 72.73)	(67.22, 72.24)

Table 4.7 The Simulation Mean and Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and $\hat{\mu}_{raj}$ for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement and the Blue-Winged Teal Population Consists of $N=5$ PSUs.

m	$\bar{\mu}_{unit}$	$\bar{\mu}_{net}$	$\bar{\mu}_{clus}$	$\bar{\mu}_{raj}$	$v(\hat{\mu}_{unit})$	$v(\hat{\mu}_{net})$	$v(\hat{\mu}_{clus})$	$v(\hat{\mu}_{raj})$
1 PSU+ 1 SSU	72.85	71.07	69.54	72.72	23,165.59	11,249.68	9,942.71	77,867.30
1 PSU+ 2 SSUs	70.33	70.94	69.48	70.56	18,473.39	9,759.01	8,610.58	64,635.78
1 PSU+ 3 SSUs	70.66	71.57	70.29	72.35	15,300.38	8,034.44	7,036.08	55,651.24
1 PSU+ 4 SSUs	69.81	71.54	70.38	71.69	12,989.21	7,115.62	5970.44	47,281.57
1 PSU+ 5 SSUs	69.42	71.56	71.06	69.53	11,116.47	6,453.38	5,278.41	39,330.17

Table 4.8 95% CI of μ That is Produced by the Simulation Mean and Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and $\hat{\mu}_{raj}$ When the Blue-Winged Teal Population Consists of $N=5$ PSUs.

m	$\bar{\mu}_{unit} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{units})}{r}}$	$\bar{\mu}_{net} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{net})}{r}}$	$\bar{\mu}_{clus} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{clus})}{r}}$	$\bar{\mu}_{raj} \pm 1.96\sqrt{\frac{v(\hat{\mu}_{raj})}{r}}$
1 PSU+ 1 SSU	(71.14, 74.58)	(69.87, 72.27)	(68.41, 70.66)	(69.57, 75.88)
1 PSU+ 2 SSUs	(68.80, 71.88)	(69.83, 72.06)	(68.43, 70.52)	(67.68, 73.43)
1 PSU+ 3 SSUs	(69.26, 72.06)	(70.55, 72.58)	(69.34, 71.24)	(69.68, 75.01)
1 PSU+ 4 SSUs	(68.52, 71.10)	(70.59, 72.49)	(69.50, 71.25)	(69.23, 74.15)
1 PSU+ 5 SSUs	(68.23, 70.61)	(70.65, 72.46)	(70.24, 71.88)	(67.29, 71.77)

Table 4.9 The Simulation Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ and Their Efficiencies for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement When the Blue-Winged Teal Population Consists of $N=10$ PSUs.

m	$v(\hat{\mu}_{unit})$	$v(\hat{\mu}_{net})$	$v(\hat{\mu}_{clus})$	$v(\hat{\mu}_{raj})$	$eff(\hat{\mu}_{unit})$	$eff(\hat{\mu}_{net})$	$eff(\hat{\mu}_{clus})$
1 PSU+ 1 SSU	30,314.75	22,264.45	21,041.99	97,970.30	3.23	4.40	4.66
1 PSU+ 2 SSUs	24,891.37	19,165.14	16,962.59	83,024.61	3.34	4.33	4.89
1 PSU+ 3 SSUs	19,606.92	15,749.96	14,452.13	66,727.47	3.40	4.24	4.62
1 PSU+ 4 SSUs	16,871.81	13,616.20	12,026.51	57,676.74	3.42	4.24	4.79
1 PSU+ 5 SSUs	14,507.58	11,911.05	10,209.43	49,309.03	3.39	4.14	4.83

Table 4.10 The Simulation Variance of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ and Their Efficiencies for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement When the Blue-Winged Teal Population Consists of $N=5$ PSUs.

m	$v(\hat{\mu}_{unit})$	$v(\hat{\mu}_{net})$	$v(\hat{\mu}_{clus})$	$v(\hat{\mu}_{raj})$	$eff(\hat{\mu}_{unit})$	$eff(\hat{\mu}_{net})$	$eff(\hat{\mu}_{clus})$
1 PSU+ 1 SSU	23,165.59	11,249.68	9,942.71	77,867.30	3.36	6.92	7.83
1 PSU+ 2 SSUs	18,473.39	9,759.01	8,610.58	64,635.78	3.49	6.62	7.51
1 PSU+ 3 SSUs	15,300.38	8,034.44	7,036.08	55,651.24	3.64	6.93	7.91
1 PSU+ 4 SSUs	12,989.21	7,115.62	5970.44	47,281.57	3.64	6.64	7.92
1 PSU+ 5 SSUs	11,116.47	6,453.38	5,278.41	39,330.17	3.54	6.09	7.45

Table 4.11 The Simulation Mean of $\hat{v}(\hat{\mu}_{unit})$, $\hat{v}(\hat{\mu}_{net})$, $\hat{v}(\hat{\mu}_{clus})$ and $\hat{v}(\hat{\mu}_{raj})$ for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

m	$\bar{v}(\hat{\mu}_{unit})$	$\bar{v}(\hat{\mu}_{net})$	$\bar{v}(\hat{\mu}_{clus})$	$\bar{v}(\hat{\mu}_{raj})$
1 PSU+ 1 SSU	29,742.71	22,115.98	20,768.48	97.439.59
1 PSU+ 2 SSUs	24,372.76	18,551.43	16,790.17	83,138.22
1 PSU+ 3 SSUs	19,678.05	14,755.56	14,321.41	68,127.99
1 PSU+ 4 SSUs	17,441.03	11,993.16	11,966.72	59,067.81
1 PSU+ 5 SSUs	14,436.05	10,338.64	10,239.39	49,897.83

Table 4.12 The Simulation Mean of $\hat{v}(\hat{\mu}_{unit})$, $\hat{v}(\hat{\mu}_{net})$, $\hat{v}(\hat{\mu}_{clus})$ and $\hat{v}(\hat{\mu}_{raj})$ for Design of Partially SACS with Units, Networks and Clusters Selected Without Replacement and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

m	$\bar{v}(\hat{\mu}_{unit})$	$\bar{v}(\hat{\mu}_{net})$	$\bar{v}(\hat{\mu}_{clus})$	$\bar{v}(\hat{\mu}_{raj})$
1 PSU+ 1 SSU	23,331.81	11,308.52	9,793.34	77,957.15
1 PSU+ 2 SSUs	18,815.30	9,432.87	8,539.38	65,428.11
1 PSU+ 3 SSUs	15,669.96	7,651.89	7,012.39	57,234.49
1 PSU+ 4 SSUs	13,053.87	6,401.57	5,967.03	48,208.70
1 PSU+ 5 SSUs	11,242.81	5,512.61	5,291.22	40,134.44

For the ability to construct 95% confidence interval of μ, the relative frequency of coverage μ of 95% confidence intervals that is produced by each estimator of $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$, $\hat{\mu}_{clus}$ and $\hat{\mu}_{raj}$ and its corresponding variance estimator are increased when the number of initial samples size is increased (see table 4.13). The relative frequency of coverage μ of 95% confidence intervals that is formed by estimator $\hat{\mu}_{clus}$ is highest of the four estimators when $N = 10$, $m =$ 1PSUs+5 SSUs and $N = 5$, $m =$ 1PSUs+5 SSUs, these are about 44 percent and 68 percent, respectively. However, their ability is less than 95 percent. As we see in figure 4.3 and figure 4.4, the histogram indicates that the distributions of these estimators are skewed. This means that their distribution do not asymptotic to normal distribution (see more details in appendix).

Table 4.13 Coverage Relative Frequencies for Confidence Intervals That are Formed by Using $\hat{\mu}_{(\bullet)i} \pm t_{\frac{\alpha}{2},(m-1)} \sqrt{\hat{v}\left(\hat{\mu}_{(\bullet)i}\right)}$.

N	m	Coverage relative frequencies of CI containing μ			
		$CI(\hat{\mu}_{unit})$	$CI(\hat{\mu}_{net})$	$CI(\hat{\mu}_{clus})$	$CI(\hat{\mu}_{raj})$
10	1 PSU+ 1 SSU	0.3481	0.3418	0.3407	0.2431
	1 PSU+ 2 SSUs	0.4013	0.3903	0.3849	0.2635
	1 PSU+ 3 SSUs	0.3815	0.3793	0.3818	0.2795
	1 PSU+ 4 SSUs	0.4089	0.3949	0.4030	0.2866
	1 PSU+ 5 SSUs	0.4333	0.4299	0.4417	0.2707
5	1 PSU+ 1 SSU	0.6339	0.6289	0.6280	0.4305
	1 PSU+ 2 SSUs	0.6513	0.6513	0.6450	0.4396
	1 PSU+ 3 SSUs	0.6558	0.6558	0.6548	0.4560
	1 PSU+ 4 SSUs	0.6630	0.6630	0.6640	0.4475
	1 PSU+ 5 SSUs	0.3732	0.6704	0.6775	0.4417

Note: $u_i\left(\hat{\mu}_{(\bullet)}\right)$ to be 1 if the 95% CI that is produced by $\hat{\mu}_{(\bullet)i} \pm t_{\frac{\alpha}{2},(m-1)} \sqrt{\hat{v}\left(\hat{\mu}_{(\bullet)i}\right)}$ contains the population mean, and 0 if the 95% CI does not.

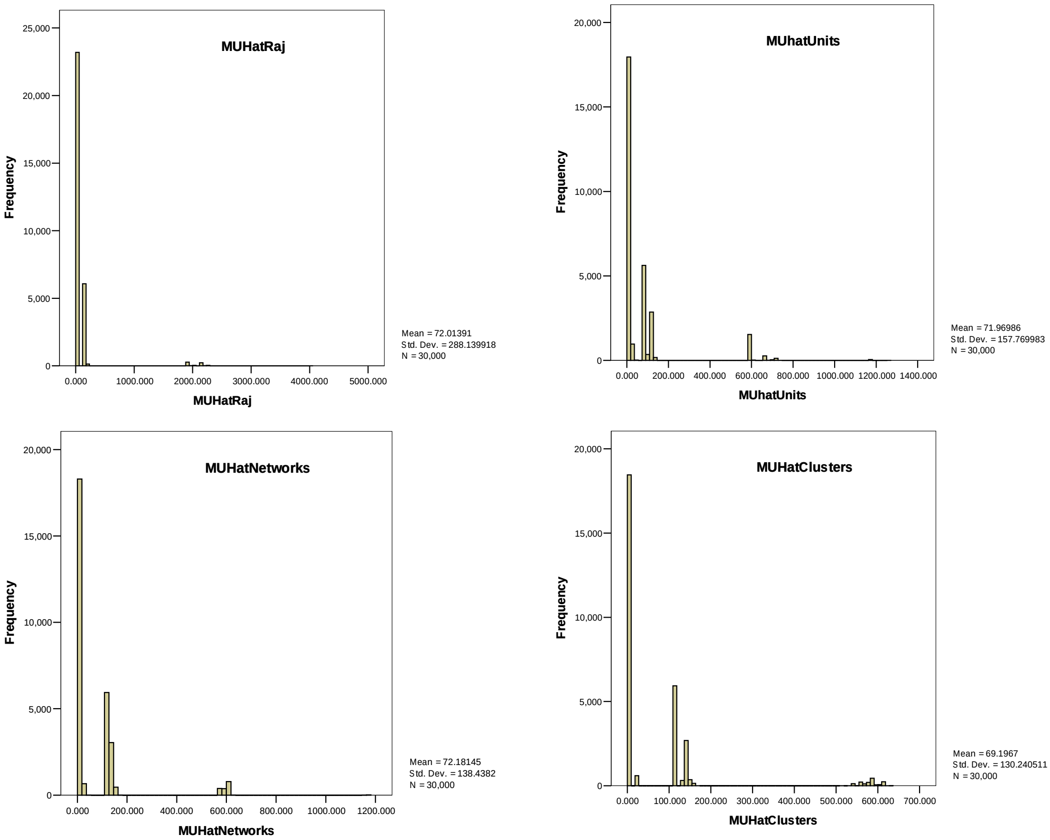


Figure 4.3 An Example of a Histogram for Estimators $\hat{\mu}_{raj}$, $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ * When the Initial Sample of Size is 1 PSU and 2 SSUs and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

Note: * MUHatRaj = $\hat{\mu}_{raj}$, MUHatRaj = $\hat{\mu}_{unit}$, MUHatNetworks = $\hat{\mu}_{net}$ and MUHatClusters = $\hat{\mu}_{clus}$.

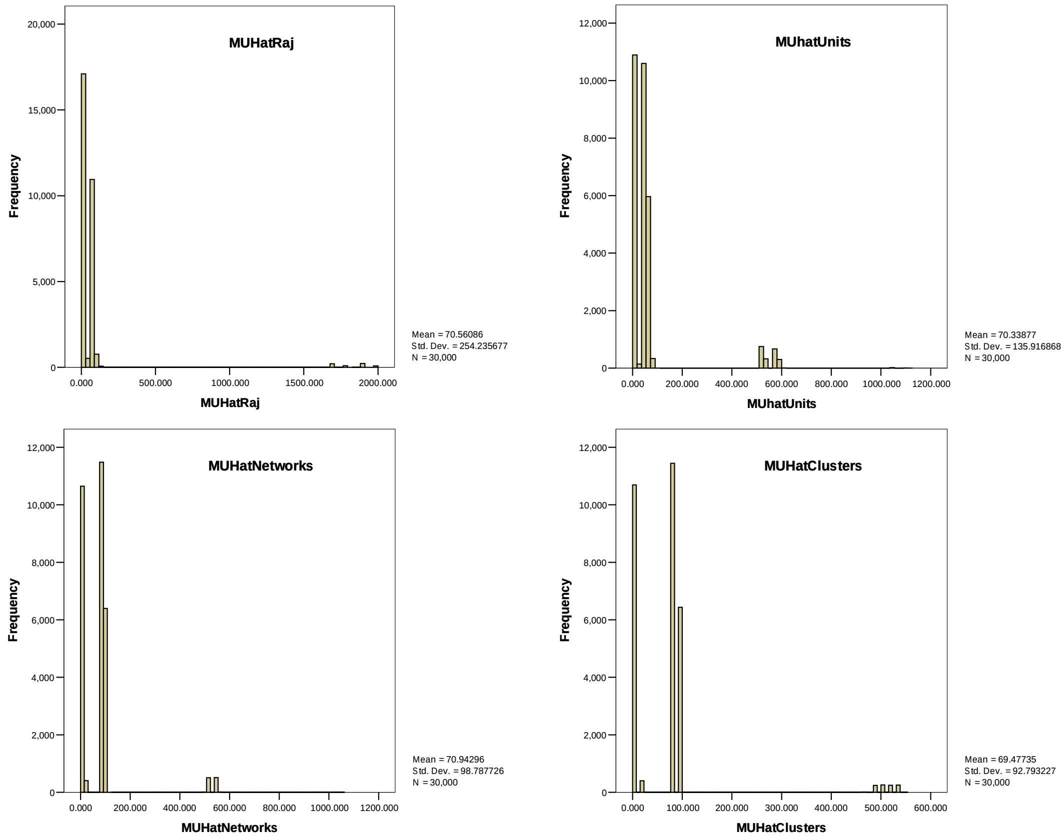


Figure 4.4 An Example of a Histogram for Estimators $\hat{\mu}_{raj}$, $\hat{\mu}_{unit}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ * When the Initial Sample of Size is 1 PSU and 2 SSUs and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

Note: * MUHatRaj = $\hat{\mu}_{raj}$, MUHatRaj = $\hat{\mu}_{unit}$, MUHatNetworks = $\hat{\mu}_{net}$ and MUHatClusters = $\hat{\mu}_{clus}$.

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

5.1 Conclusion

This research is composed of two topics in the area of variance estimation in adaptive cluster sampling when a single primary sampling unit is selected and the design of partially systematic adaptive cluster sampling.

5.1.1 The Conclusions for the Variance Estimation in Adaptive Cluster Sampling

For adaptive cluster sampling, (ACS) with a single primary sampling unit (PSU) is chosen, and two new biased variance estimators were proposed according to concepts of splitting the initial sample into sub-samples ($\hat{v}_{acs(1)}(\hat{\mu})$) and treating the initial sample as if it were a stratified sample ($\hat{v}_{acs(2)}(\hat{\mu})$). The simulation study showed that both new variance estimators were underestimated while non-adaptive estimators were overestimated. The estimator $\hat{v}_{acs(1)}(\hat{\mu})$ was not preferable when the number of sub-samples equaled two and the population consists of 10 primary sampling units because its relative bias was too large for 46.49 percentage of the true variance. However, the relative bias of $\hat{v}_{acs(1)}(\hat{\mu})$ was decreased when the number of sub-samples (p) was increased. The efficiency comparison between these two new variance estimators was made when the size of sub-samples or a stratum size was two. From this simulation, it was seen that the estimator $\hat{v}_{acs(2)}(\hat{\mu})$ seems to be a good choice because it had smaller relative bias and mean squared error than the first. The property comparison between the estimator $\hat{v}_{acs(1)}(\hat{\mu})$ and non-adaptive estimator $\hat{v}_{sys(1)}(\bar{y})$ was performed for several values of p, these are p =2, 4, 5, 10 when population consists of 10 primary sampling units and p =2, 4, 5, 8, 10, 20 when population consists of 5 primary sampling units. The estimator $\hat{v}_{acs(1)}(\hat{\mu})$ was less efficient than non-adaptive estimator in terms of minimum relative bias for all cases of p when the population consisted of 10 PSUs but was more efficient than non-adaptive estimator for all cases of p when the population consisted of 5 PSUs, respectively. The comparison result between estimator $\hat{v}_{acs(2)}(\hat{\mu})$ and non-adaptive estimator $\hat{v}_{sys(2)}(\bar{y})$ indicated that estimator $\hat{v}_{acs(2)}(\hat{\mu})$ was more efficient in terms of minimum relative bias than non-adaptive estimator when the number of sub-samples equaled 20 and population consists of 5 primary sampling units but was less efficient than non-adaptive estimator when the number of sub-samples equaled 10 and population consists of 10 primary sampling units. Though the frequency relative of coverage population mean of 95% confidence interval that is formed by new variance estimators was higher than of non-adaptive estimators, that was 0.6 or 60 percent, but less than 0.95 or 95 percent. However, the proposed variance estimators did not use information of edge units and ignored the correlation terms between mean of sub-samples or units within sub-samples or strata. This information may reduce their relative bias.

5.1.2 The Conclusions for the Partially Systematic Adaptive Cluster Sampling

For partially systematic adaptive cluster sampling (partially SACS), three sampling procedures for a selection without replacement of units (based on Raj, 1956: 269-284), without replacement of networks (based on Salehi and Seber, 1997: 209-219; Dryver and Thompson, 2007: 35-43), and without replacement of clusters (based on Dryver and Thompson, 2007: 35-43) were used to select the initial sample. All three sampling procedures can provide unbiased estimator of the population mean and variance. An efficiency comparison between these three unbiased estimators proposed for the population mean was performed, using simulation, for several values of the initial sample size. The simulation study showed that an unbiased estimator of the population mean based on a selection without replacement of cluster was the most efficient in terms of minimum variance, while an unbiased estimator based on a selection without replacement of units was the least efficient because its variance was larger than the variance of other estimators. In addition, the efficiency of these unbiased estimators of the population mean gained more when the size of initial sample was increased. When these unbiased estimators of population mean were compared to unbiased estimator of population mean proposed by Raj (1956: 269-284), it was seen that unbiased estimators of the population mean proposed for partially SACS were more efficient than for unbiased estimator proposed by Raj or non-adaptive estimator. The intervals containing the population mean was a 95% confidence interval performed using each estimator of the population mean and the variance. The results indicated that the relative frequency of coverage population mean of 95% confidence interval that is formed by each estimator of population mean and variance was increased when the initial sample size was increased. However, the relative frequency of coverage population mean for each estimator was less than 0.95 or 95 percent. This may have resulted from the fact that the distribution of the proposed estimators and non-adaptive estimator of the population mean was not asymptotic to normal distribution.

5.2 Future Research

For adaptive cluster sampling with a single primary sampling unit, further study is required in order to improve the proposed variance estimators by using information of edge units or considered in terms of correlation between sub-samples or units within stratum. Another point is to find other alternative variance estimators based on another concept of estimation, such as under the condition of a population with random effect, etc, and to develop a variance estimator based on other issues, such as the modified Hajeck-variance estimator.

For partially systematic adaptive cluster sampling, further study is needed to improve the proposed estimators by using the Rao-Blackwell method. Another point is that the efficiency comparison between the proposed estimators and another estimator begins with for a multiple random start in systematic sampling or systematic adaptive cluster sampling.

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APPENDIX

A Histogram for Proposed Estimators of the Population Mean in Chapter 4

The histogram for estimators proposed in chapter 4 is shown in this part. For $N = 5$, the results is shown in figure A1 to A4. The results indicate that the distributions of these estimators are skewed.

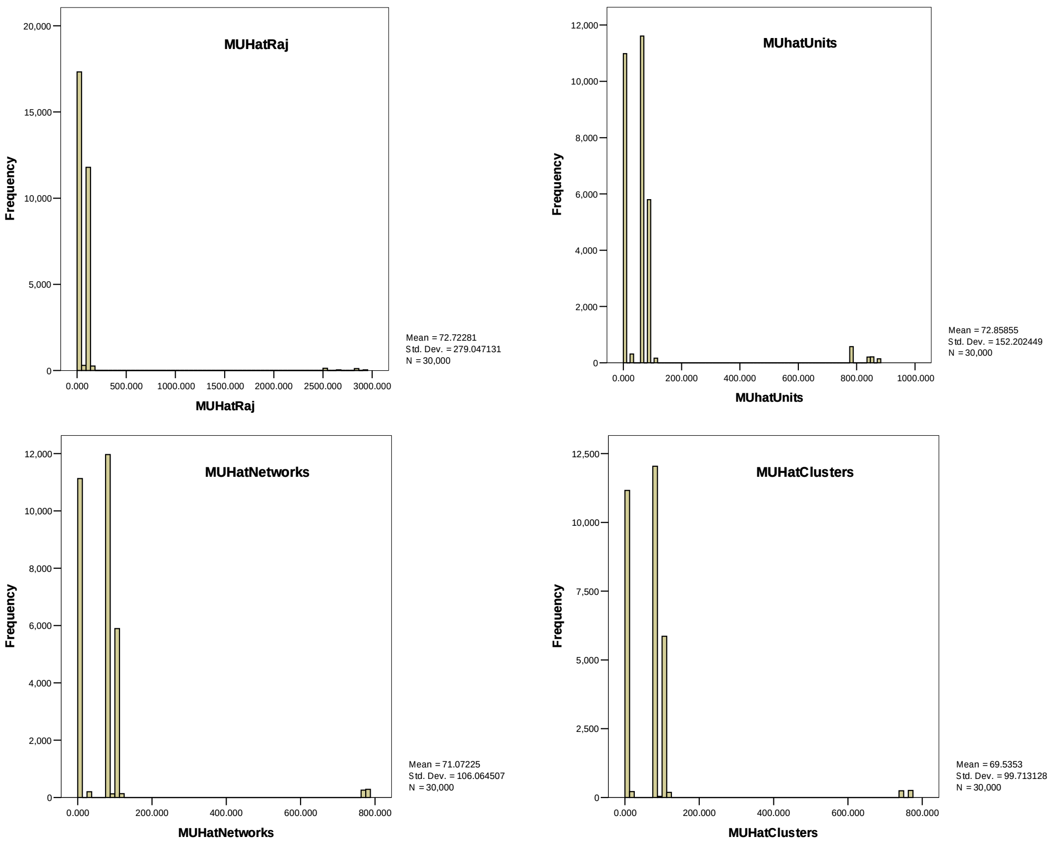

Figure A1 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 1 SSU and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

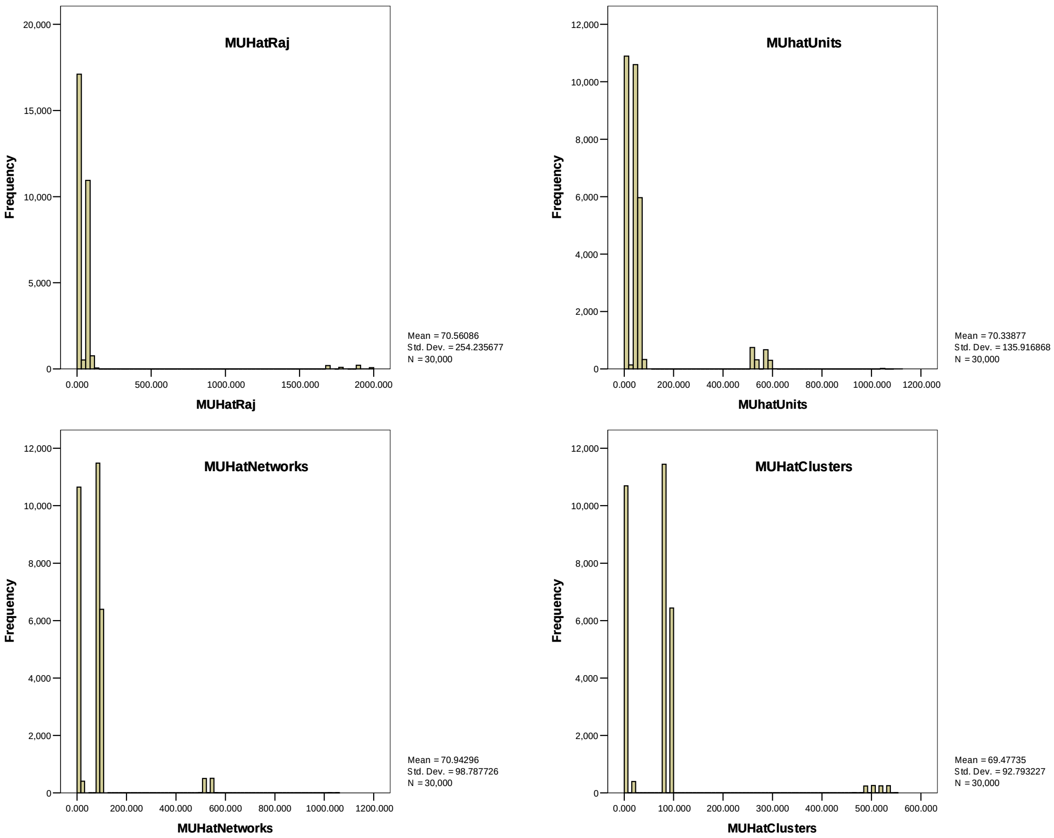

Figure A2 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 2 SSUs and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

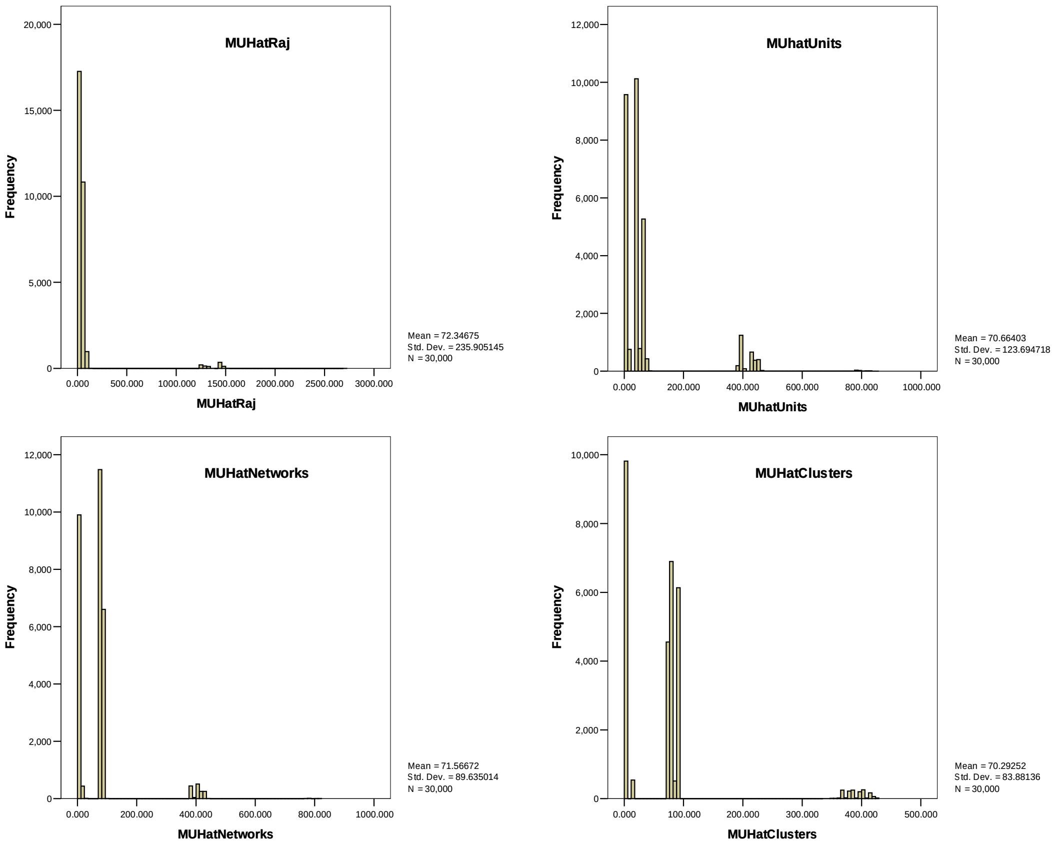

Figure A3 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 3 SSUs and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

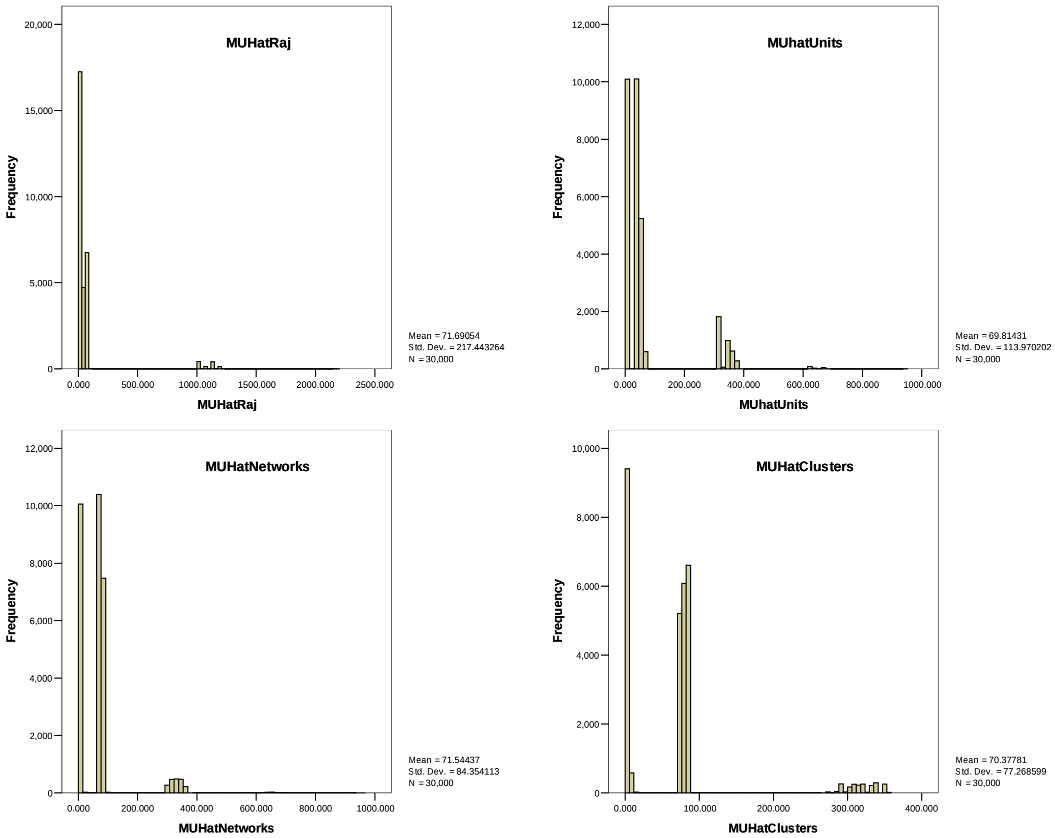

Figure A4 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 4 SSUs and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

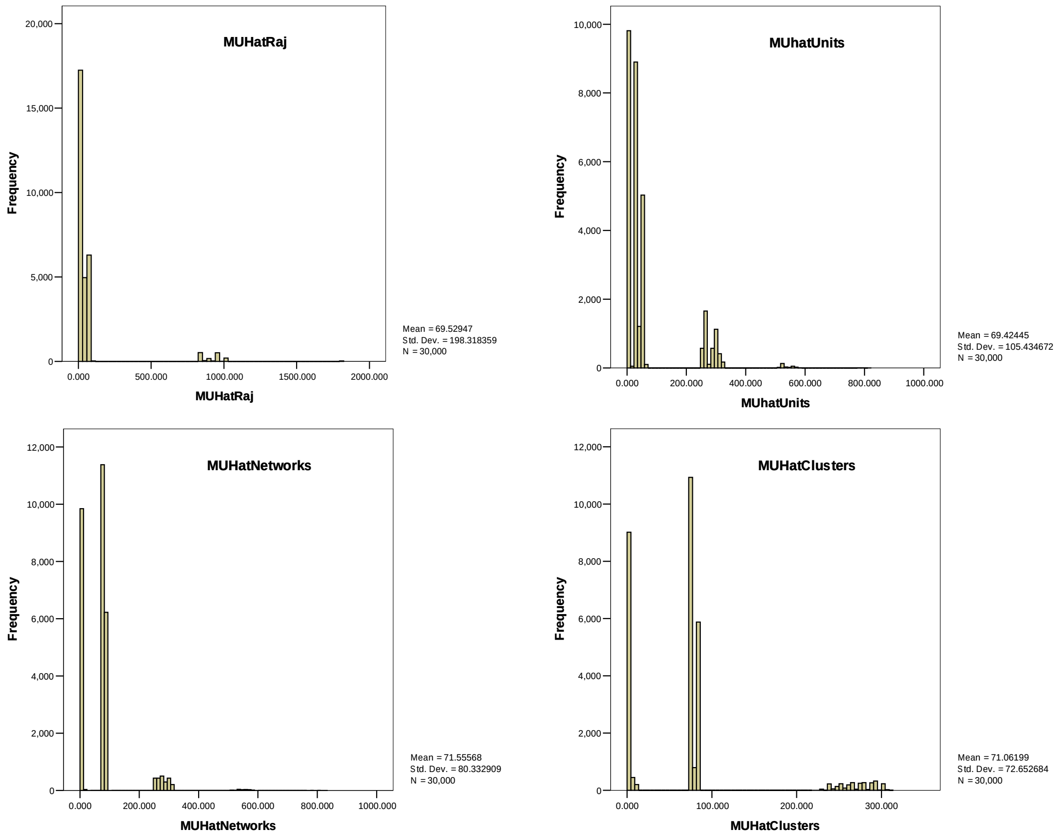

Figure A5 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 5 SSUs and the Blue-Winged Teal Population Consists of $N = 5$ PSUs.

For $N = 10$, a histogram of proposed estimators of the population mean is shown in figure A6 to A9.

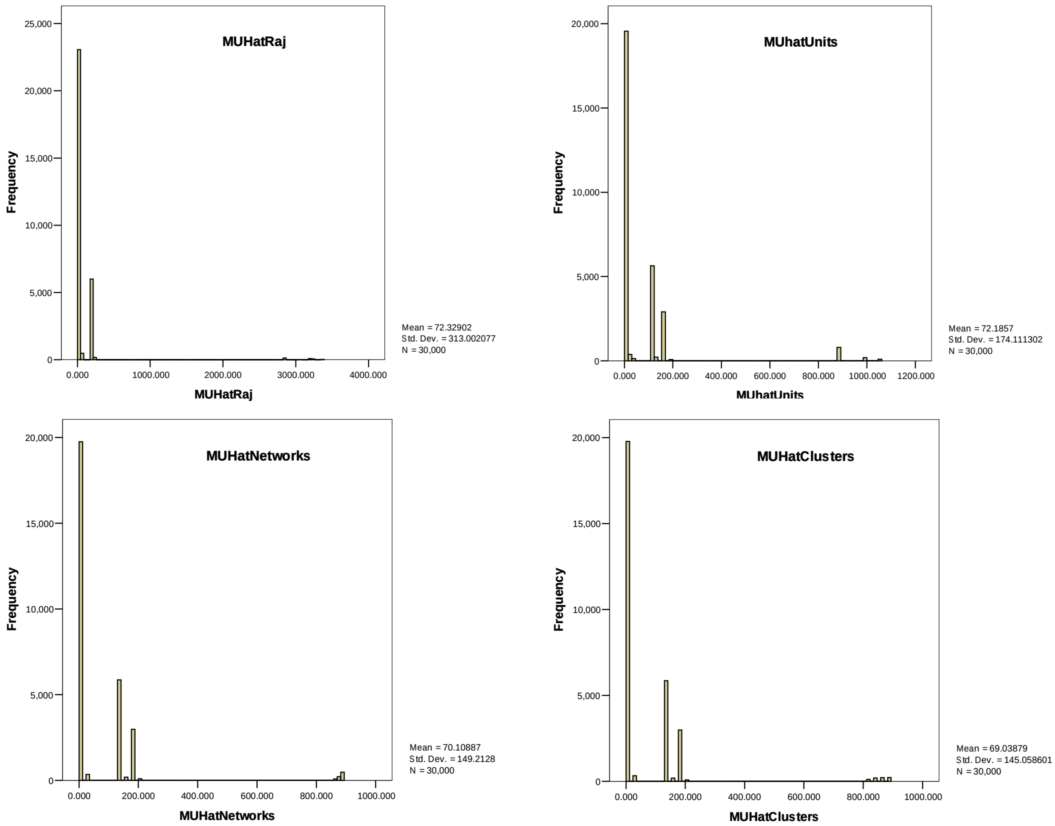

Figure A6 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 1 SSU and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

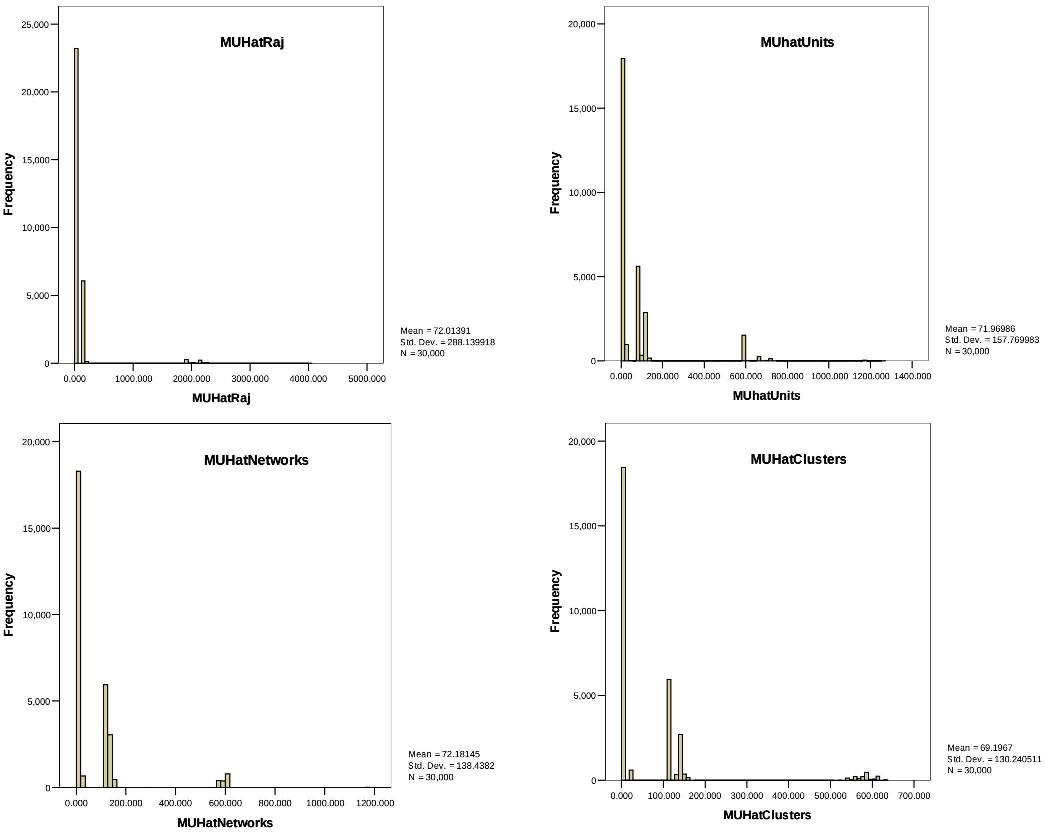

Figure A7 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 2 SSUs and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

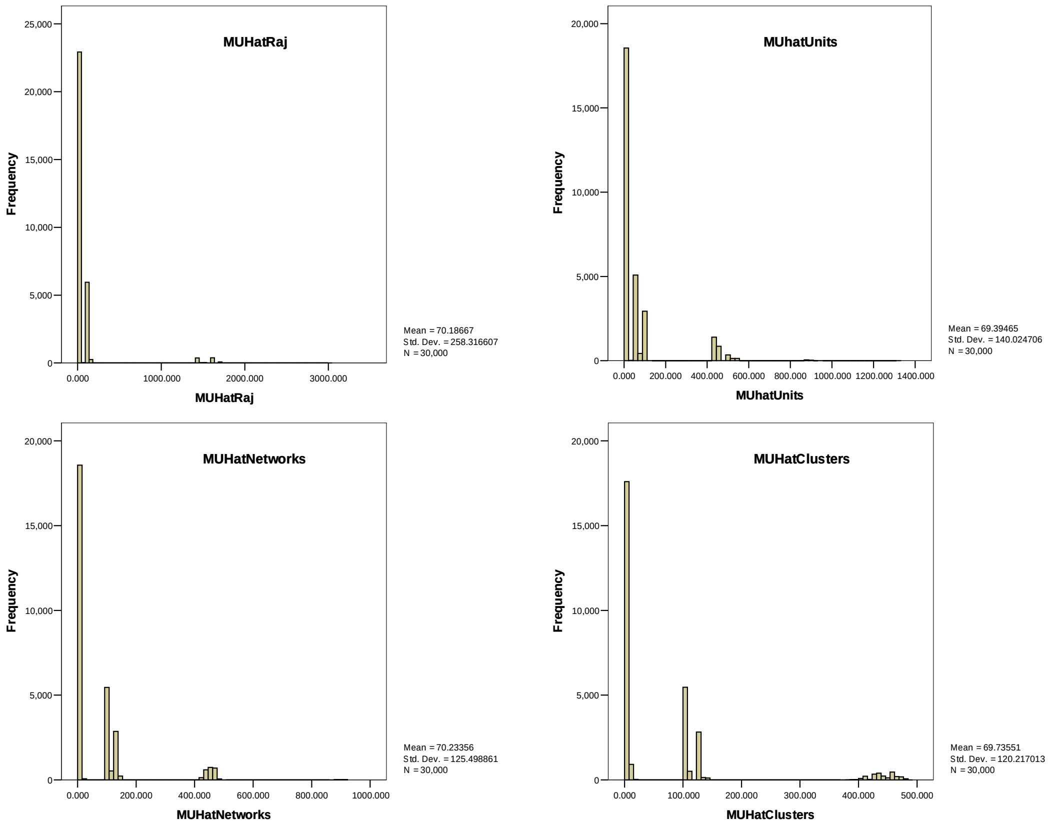

Figure A8 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 3 SSUs and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

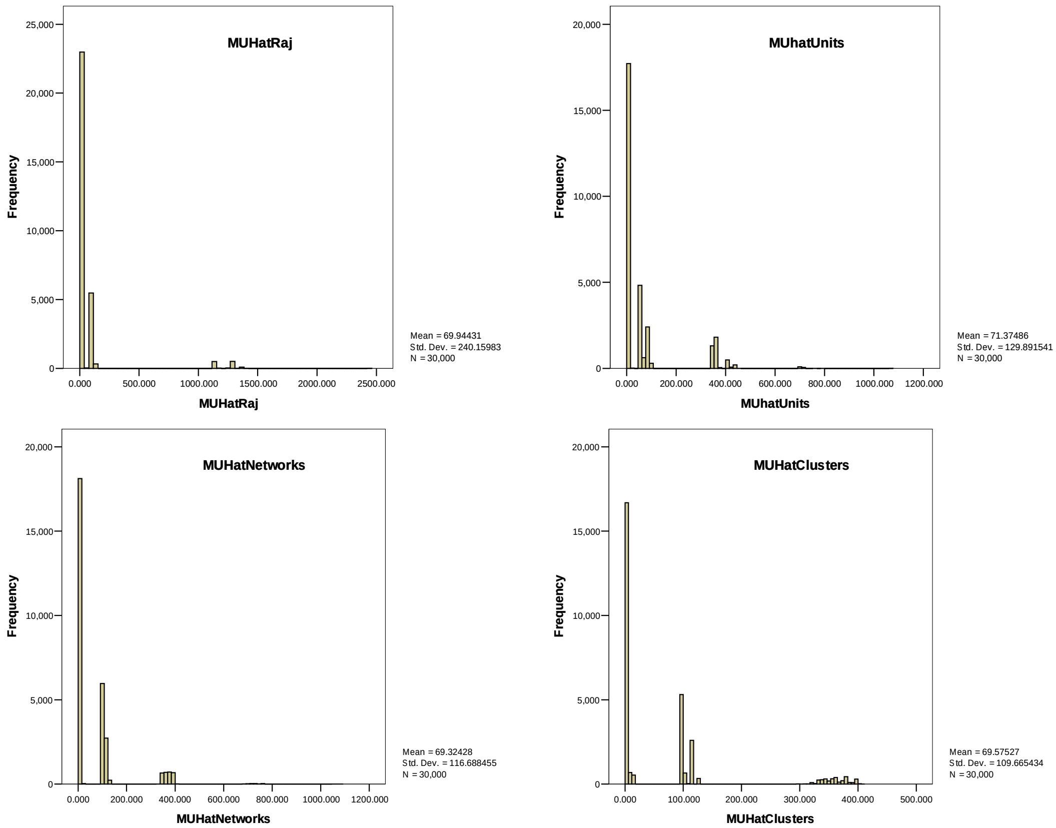

Figure A9 A Histogram for Estimators $\hat{\mu}_{unit}$, $\hat{\mu}_{raj}$, $\hat{\mu}_{net}$ and $\hat{\mu}_{clus}$ When the Initial Sample of Size is 1 PSU and 4 SSUs and the Blue-Winged Teal Population Consists of $N = 10$ PSUs.

BIOGRAPHY

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